

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.3
Miscellaneous"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 12: Result is not expressed in closed-form.

$$\int \frac{1}{3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3} dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/3} + \frac{2(b+cx)}{(b^2-3ac)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} b^{2/3} (b^2 - 3ac)^{2/3}} + \frac{\text{Log}\left[b - b^{1/3} (b^2 - 3ac)^{1/3} + cx\right]}{3 b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\text{Log}\left[b^{2/3} (b^2 - 3ac)^{2/3} + b^{1/3} c (b^2 - 3ac)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{6 b^{2/3} (b^2 - 3ac)^{2/3}}$$

Result (type 7, 63 leaves):

$$\frac{1}{3} \text{RootSum}\left[3 a b + 3 b^2 \#1 + 3 b c \#1^2 + c^2 \#1^3 \&, \frac{\text{Log}[x - \#1]}{b^2 + 2 b c \#1 + c^2 \#1^2} \&\right]$$

Problem 13: Result is not expressed in closed-form.

$$\int \frac{1}{(3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^2} dx$$

Optimal (type 3, 245 leaves, 8 steps):

$$-\frac{c \left(\frac{b}{c} + x\right)}{3 b (b^2 - 3 a c) (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)} + \frac{2 c \operatorname{ArcTan}\left[\frac{b^{1/3} + \frac{2(b+c)x}{(b^2-3ac)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{3 \sqrt{3} b^{5/3} (b^2 - 3 a c)^{5/3}} -$$

$$\frac{2 c \operatorname{Log}\left[b - b^{1/3} (b^2 - 3 a c)^{1/3} + c x\right]}{9 b^{5/3} (b^2 - 3 a c)^{5/3}} + \frac{c \operatorname{Log}\left[b^{2/3} (b^2 - 3 a c)^{2/3} + b^{1/3} c (b^2 - 3 a c)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{9 b^{5/3} (b^2 - 3 a c)^{5/3}}$$

Result (type 7, 112 leaves):

$$-\frac{\frac{3(b+c x)}{3 a b+x(3 b^2+3 b c x+c^2 x^2)} + 2 c \operatorname{RootSum}\left[3 a b+3 b^2 \#1+3 b c \#1^2+c^2 \#1^3 \&, \frac{\operatorname{Log}[x-\#1]}{b^2+2 b c \#1+c^2 \#1^2} \&\right]}{9 (b^3-3 a b c)}$$

Problem 14: Result is not expressed in closed-form.

$$\int \frac{1}{(3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^3} dx$$

Optimal (type 3, 305 leaves, 9 steps):

$$-\frac{c \left(\frac{b}{c} + x\right)}{6 b (b^2 - 3 a c) (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^2} + \frac{5 c^2 \left(\frac{b}{c} + x\right)}{18 b^2 (b^2 - 3 a c)^2 (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)} - \frac{5 c^2 \operatorname{ArcTan}\left[\frac{b^{1/3} + \frac{2(b+c)x}{(b^2-3ac)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{9 \sqrt{3} b^{8/3} (b^2 - 3 a c)^{8/3}} +$$

$$\frac{5 c^2 \operatorname{Log}\left[b - b^{1/3} (b^2 - 3 a c)^{1/3} + c x\right]}{27 b^{8/3} (b^2 - 3 a c)^{8/3}} - \frac{5 c^2 \operatorname{Log}\left[b^{2/3} (b^2 - 3 a c)^{2/3} + b^{1/3} c (b^2 - 3 a c)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{54 b^{8/3} (b^2 - 3 a c)^{8/3}}$$

Result (type 7, 149 leaves):

$$\frac{1}{54 (b^3 - 3 a b c)^2}$$

$$\left(-\frac{3 (b+c x) (3 b^3 - 15 b^2 c x - 5 c^3 x^3 - 3 b c (8 a + 5 c x^2))}{(3 a b+x(3 b^2+3 b c x+c^2 x^2))^2} + 10 c^2 \operatorname{RootSum}\left[3 a b+3 b^2 \#1+3 b c \#1^2+c^2 \#1^3 \&, \frac{\operatorname{Log}[x-\#1]}{b^2+2 b c \#1+c^2 \#1^2} \&\right] \right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int (b x + c x^2 + d x^3)^n dx$$

Optimal (type 6, 132 leaves, 3 steps):

$$\frac{1}{1+n} x \left(1 + \frac{2dx}{c - \sqrt{c^2 - 4bd}}\right)^{-n} \left(1 + \frac{2dx}{c + \sqrt{c^2 - 4bd}}\right)^{-n} (bx + cx^2 + dx^3)^n \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2dx}{c - \sqrt{c^2 - 4bd}}, -\frac{2dx}{c + \sqrt{c^2 - 4bd}}\right]$$

Result (type 6, 438 leaves):

$$\begin{aligned} & \left(2^{-1-n} d \left(c + \sqrt{c^2 - 4bd}\right) (2+n) x^2 \left(\frac{c - \sqrt{c^2 - 4bd}}{2d} + x\right)^{-n} \left(\frac{c - \sqrt{c^2 - 4bd} + 2dx}{d}\right)^{1+n} \right. \\ & \left. \left(2b + \left(c - \sqrt{c^2 - 4bd}\right) x\right)^2 \left(x(b + x(c + dx))\right)^{-1+n} \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{-c + \sqrt{c^2 - 4bd}}\right] \right) / \\ & \left(\left(-c + \sqrt{c^2 - 4bd}\right) (1+n) \left(c + \sqrt{c^2 - 4bd} + 2dx\right) \left(-2b(2+n) \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{-c + \sqrt{c^2 - 4bd}}\right] + \right. \right. \\ & \left. \left. n x \left(\left(-c + \sqrt{c^2 - 4bd}\right) \text{AppellF1}\left[2+n, 1-n, -n, 3+n, -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{-c + \sqrt{c^2 - 4bd}}\right] - \right. \right. \right. \\ & \left. \left. \left. \left(c + \sqrt{c^2 - 4bd}\right) \text{AppellF1}\left[2+n, -n, 1-n, 3+n, -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{-c + \sqrt{c^2 - 4bd}}\right] \right) \right) \right) \end{aligned}$$

Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + dx^3)^n dx$$

Optimal (type 5, 35 leaves, 2 steps):

$$\frac{x (a + dx^3)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{4}{3} + n, \frac{4}{3}, -\frac{dx^3}{a}\right]}{a}$$

Result (type 6, 196 leaves):

$$\begin{aligned} & \frac{1}{d^{1/3} (1+n)} 2^{-n} \left((-1)^{2/3} a^{1/3} + d^{1/3} x\right) \left(\frac{a^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}\right)^{-n} \left(\frac{i \left(1 + \frac{d^{1/3} x}{a^{1/3}}\right)}{3i + \sqrt{3}}\right)^{-n} \\ & (a + dx^3)^n \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{i \left((-1)^{2/3} a^{1/3} + d^{1/3} x\right)}{\sqrt{3} a^{1/3}}, \frac{i + \sqrt{3} - \frac{2i d^{1/3} x}{a^{1/3}}}{3i + \sqrt{3}}\right] \end{aligned}$$

Problem 37: Result is not expressed in closed-form.

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} dx$$

Optimal (type 3, 529 leaves, 10 steps):

$$\begin{aligned}
 & \frac{d \operatorname{ArcTanh} \left[\frac{\sqrt{2} c + c^{3/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} + \sqrt{2} d x}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} \right]}{2 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} + \frac{d \operatorname{ArcTanh} \left[\frac{c^{1/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} - \sqrt{2} (c + d x)}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} \right]}{2 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} - \\
 & \frac{d \operatorname{Log} \left[\sqrt{c} \sqrt{c^3 + 4 a d^2} - \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right]}{4 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}} + \\
 & \frac{d \operatorname{Log} \left[\sqrt{c} \sqrt{c^3 + 4 a d^2} + \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right]}{4 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}}
 \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{1}{4} \operatorname{RootSum} \left[4 a c + 4 c^2 \#1^2 + 4 c d \#1^3 + d^2 \#1^4 \&, \frac{\operatorname{Log} [x - \#1]}{2 c^2 \#1 + 3 c d \#1^2 + d^2 \#1^3} \& \right]$$

Problem 38: Result is not expressed in closed-form.

$$\int \frac{1}{(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^2} dx$$

Optimal (type 3, 746 leaves, 11 steps):

$$\begin{aligned}
& - \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4 a d^2 - c d^2 \left(\frac{c}{d} + x\right)^2\right)}{16 a c \left(c^3 + 4 a d^2\right) \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4\right)} - \frac{d \left(c^3 + 12 a d^2 + c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{2} c + c^{1/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} + \sqrt{2} d x}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}}\right]}{32 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} + \\
& \frac{d \left(c^3 + 12 a d^2 + c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} - \sqrt{2} (c + d x)}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}}\right]}{32 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} - \\
& \left(d \left(c^3 + 12 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{Log}\left[\sqrt{c} \sqrt{c^3 + 4 a d^2} - \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x\right) + d^2 \left(\frac{c}{d} + x\right)^2\right] \right) / \\
& \left(64 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \right) + \\
& \left(d \left(c^3 + 12 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{Log}\left[\sqrt{c} \sqrt{c^3 + 4 a d^2} + \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x\right) + d^2 \left(\frac{c}{d} + x\right)^2\right] \right) / \\
& \left(64 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \right)
\end{aligned}$$

Result (type 7, 182 leaves):

$$\begin{aligned}
& \frac{1}{64 a c \left(c^3 + 4 a d^2\right)} \left(\frac{4 \left(c + d x\right) \left(4 a d + c x\right) \left(2 c + d x\right)}{4 a c + x^2 \left(2 c + d x\right)^2} + \right. \\
& \left. \operatorname{RootSum}\left[4 a c + 4 c^2 \#1^2 + 4 c d \#1^3 + d^2 \#1^4 \&, \frac{2 c^3 \operatorname{Log}\left[x - \#1\right] + 12 a d^2 \operatorname{Log}\left[x - \#1\right] + 2 c^2 d \operatorname{Log}\left[x - \#1\right] \#1 + c d^2 \operatorname{Log}\left[x - \#1\right] \#1^2}{2 c^2 \#1 + 3 c d \#1^2 + d^2 \#1^3} \&\right] \right)
\end{aligned}$$

Problem 43: Result is not expressed in closed-form.

$$\int \frac{1}{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right]}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2 \operatorname{ArcTanh}\left[\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right]}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

Result (type 7, 71 leaves):

$$-\operatorname{RootSum}\left[8ae^2 - d^3\#1 + 8de^2\#1^3 + 8e^3\#1^4 \&, \frac{\operatorname{Log}[x - \#1]}{d^3 - 24de^2\#1^2 - 32e^3\#1^3} \&\right]$$

Problem 44: Result is not expressed in closed-form.

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx$$

Optimal (type 3, 342 leaves, 5 steps):

$$\frac{2e\left(\frac{d}{4e} + x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} -$$

$$\frac{24e\left(d^4 + 128ae^3 - d^2\sqrt{d^4 - 64ae^3}\right)\operatorname{ArcTanh}\left[\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right]}{(d^4 - 64ae^3)^{3/2}(5d^4 + 256ae^3)\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} + \frac{24e\left(d^4 + 128ae^3 + d^2\sqrt{d^4 - 64ae^3}\right)\operatorname{ArcTanh}\left[\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right]}{(d^4 - 64ae^3)^{3/2}(5d^4 + 256ae^3)\sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}$$

Result (type 7, 234 leaves):

$$\frac{(d+4ex)(5d^4 - 128ae^3 - 12d^3ex - 24d^2e^2x^2)}{(d^4 - 64ae^3)(5d^4 + 256ae^3)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} +$$

$$\frac{48e^2 \operatorname{RootSum}\left[8ae^2 - d^3\#1 + 8de^2\#1^3 + 8e^3\#1^4 \&, \frac{32ae^2 \operatorname{Log}[x - \#1] + d^3 \operatorname{Log}[x - \#1] \#1 + 2d^2e \operatorname{Log}[x - \#1] \#1^2}{-d^3 + 24de^2\#1^2 + 32e^3\#1^3} \&\right]}{-5d^8 + 64ad^4e^3 + 16384a^2e^6}$$

Problem 49: Result is not expressed in closed-form.

$$\int \frac{1}{8 + 8x - x^3 + 8x^4} dx$$

Optimal (type 3, 268 leaves, 16 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right]}{12\sqrt{7}} - \frac{1}{12} \sqrt{\frac{109+67\sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2-\sqrt{6(1+\sqrt{29})}+\frac{8}{x}}{\sqrt{6(-1+\sqrt{29})}}\right] - \frac{1}{12} \sqrt{\frac{109+67\sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2+\sqrt{6(1+\sqrt{29})}+\frac{8}{x}}{\sqrt{6(-1+\sqrt{29})}}\right] - \\
& \frac{1}{24} \sqrt{\frac{-109+67\sqrt{29}}{1218}} \text{Log}\left[3\sqrt{29}-\sqrt{6(1+\sqrt{29})}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right] + \frac{1}{24} \sqrt{\frac{-109+67\sqrt{29}}{1218}} \text{Log}\left[3\sqrt{29}+\sqrt{6(1+\sqrt{29})}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 45 leaves):

$$\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4 \&, \frac{\text{Log}[x-\#1]}{8-3\#1^2+32\#1^3} \&\right]$$

Problem 50: Result is not expressed in closed-form.

$$\int \frac{1}{(8+8x-x^3+8x^4)^2} dx$$

Optimal (type 3, 357 leaves, 18 steps):

$$\begin{aligned}
& - \frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{17 \text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right]}{1008\sqrt{7}} - \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2-\sqrt{6(1+\sqrt{29})}+\frac{8}{x}}{\sqrt{6(-1+\sqrt{29})}}\right]}{87696} - \\
& \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2+\sqrt{6(1+\sqrt{29})}+\frac{8}{x}}{\sqrt{6(-1+\sqrt{29})}}\right]}{87696} - \frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}} \text{Log}\left[3\sqrt{29}-\sqrt{6(1+\sqrt{29})}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]}{175392} + \\
& \frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}} \text{Log}\left[3\sqrt{29}+\sqrt{6(1+\sqrt{29})}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]}{175392}
\end{aligned}$$

Result (type 7, 113 leaves):

$$\frac{544+1539x-1146x^2+784x^3}{43848(8+8x-x^3+8x^4)} + \frac{\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4 \&, \frac{2243 \text{Log}[x-\#1]-1097 \text{Log}[x-\#1]\#1+392 \text{Log}[x-\#1]\#1^2}{8-3\#1^2+32\#1^3} \&\right]}{21924}$$

Problem 55: Result is not expressed in closed-form.

$$\int \frac{1}{1 + 4x + 4x^2 + 4x^4} dx$$

Optimal (type 3, 234 leaves, 15 steps):

$$\frac{1}{2} \operatorname{ArcTan}\left[\frac{1}{2} \left(-1 + \left(1 + \frac{1}{x}\right)^2\right)\right] - \frac{1}{2} \sqrt{\frac{1}{5} (2 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{2 - \sqrt{2(1 + \sqrt{5})} + \frac{2}{x}}{\sqrt{2(-1 + \sqrt{5})}}\right] - \frac{1}{2} \sqrt{\frac{1}{5} (2 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{2 + \sqrt{2(1 + \sqrt{5})} + \frac{2}{x}}{\sqrt{2(-1 + \sqrt{5})}}\right] -$$

$$\frac{1}{4} \sqrt{\frac{1}{5} (-2 + \sqrt{5})} \operatorname{Log}\left[\sqrt{5} - \sqrt{2(1 + \sqrt{5})} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right] + \frac{1}{4} \sqrt{\frac{1}{5} (-2 + \sqrt{5})} \operatorname{Log}\left[\sqrt{5} + \sqrt{2(1 + \sqrt{5})} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right]$$

Result (type 7, 47 leaves):

$$\frac{1}{4} \operatorname{RootSum}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4, \frac{\operatorname{Log}[x - \#1]}{1 + 2 \#1 + 4 \#1^3} \&\right]$$

Problem 56: Result is not expressed in closed-form.

$$\int \frac{1}{(1 + 4x + 4x^2 + 4x^4)^2} dx$$

Optimal (type 3, 317 leaves, 17 steps):

$$\begin{aligned}
& - \frac{17 - \left(1 + \frac{1}{x}\right)^2}{2 \left(5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right)} + \frac{\left(59 - 17 \left(1 + \frac{1}{x}\right)^2\right) \left(1 + \frac{1}{x}\right)}{10 \left(5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right)} + \frac{7}{4} \operatorname{ArcTan}\left[\frac{1}{2} \left(-1 + \left(1 + \frac{1}{x}\right)^2\right)\right] - \\
& \frac{1}{20} \sqrt{\frac{1}{10} \left(5959 + 2665 \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 - \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] - \frac{1}{20} \sqrt{\frac{1}{10} \left(5959 + 2665 \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 + \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] + \\
& \frac{1}{40} \sqrt{\frac{1}{10} \left(-5959 + 2665 \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} - \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right] - \\
& \frac{1}{40} \sqrt{\frac{1}{10} \left(-5959 + 2665 \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} + \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 108 leaves):

$$\frac{1}{40} \left(\frac{38 + 84x - 32x^2 + 72x^3}{1 + 4x + 4x^2 + 4x^4} + \operatorname{RootSum}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4, \frac{27 \operatorname{Log}[x - \#1] - 16 \operatorname{Log}[x - \#1] \#1 + 18 \operatorname{Log}[x - \#1] \#1^2}{1 + 2\#1 + 4\#1^3} \& \right] \right)$$

Problem 61: Result is not expressed in closed-form.

$$\int \frac{1}{8 + 24x + 8x^2 - 15x^3 + 8x^4} dx$$

Optimal (type 3, 263 leaves, 16 steps):

$$\begin{aligned}
& - \frac{1}{4} \sqrt{\frac{5167 + 235 \sqrt{517}}{40326}} \operatorname{ArcTan}\left[\frac{6 - \sqrt{2 \left(19 + \sqrt{517}\right)} + \frac{8}{x}}{\sqrt{2 \left(-19 + \sqrt{517}\right)}}\right] - \frac{1}{4} \sqrt{\frac{5167 + 235 \sqrt{517}}{40326}} \operatorname{ArcTan}\left[\frac{6 + \sqrt{2 \left(19 + \sqrt{517}\right)} + \frac{8}{x}}{\sqrt{2 \left(-19 + \sqrt{517}\right)}}\right] + \\
& \frac{1}{4} \sqrt{\frac{3}{13}} \operatorname{ArcTan}\left[\frac{8 + 12x - 5x^2}{\sqrt{39} x^2}\right] - \frac{1}{8} \sqrt{\frac{-5167 + 235 \sqrt{517}}{40326}} \operatorname{Log}\left[\sqrt{517} - \sqrt{2 \left(19 + \sqrt{517}\right)} \left(3 + \frac{4}{x}\right) + \left(3 + \frac{4}{x}\right)^2\right] + \\
& \frac{1}{8} \sqrt{\frac{-5167 + 235 \sqrt{517}}{40326}} \operatorname{Log}\left[\sqrt{517} + \sqrt{2 \left(19 + \sqrt{517}\right)} \left(3 + \frac{4}{x}\right) + \left(3 + \frac{4}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 55 leaves):

$$\text{RootSum}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \frac{\text{Log}[x - \#1]}{24 + 16 \#1 - 45 \#1^2 + 32 \#1^3} \&\right]$$

Problem 62: Result is not expressed in closed-form.

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx$$

Optimal (type 3, 366 leaves, 18 steps):

$$\begin{aligned} & -\frac{3 \left(3359 - 107 \left(3 + \frac{4}{x}\right)^2\right)}{208 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right)} \\ & \frac{\sqrt{\frac{19+\sqrt{517}}{40326}} \left(1678181 + 74897 \sqrt{517}\right) \text{ArcTan}\left[\frac{6-\sqrt{2(19+\sqrt{517})+\frac{8}{x}}}{\sqrt{2(-19+\sqrt{517})}}\right] - \sqrt{\frac{19+\sqrt{517}}{40326}} \left(1678181 + 74897 \sqrt{517}\right) \text{ArcTan}\left[\frac{6+\sqrt{2(19+\sqrt{517})+\frac{8}{x}}}{\sqrt{2(-19+\sqrt{517})}}\right]}{645216} + \\ & \frac{73}{208} \sqrt{\frac{3}{13}} \text{ArcTan}\left[\frac{8+12x-5x^2}{\sqrt{39}x^2}\right] - \frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}} \text{Log}\left[\sqrt{517} - \sqrt{2(19+\sqrt{517})\left(3+\frac{4}{x}\right) + \left(3+\frac{4}{x}\right)^2}\right]}{645216} + \\ & \frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}} \text{Log}\left[\sqrt{517} + \sqrt{2(19+\sqrt{517})\left(3+\frac{4}{x}\right) + \left(3+\frac{4}{x}\right)^2}\right]}{645216} \end{aligned}$$

Result (type 7, 128 leaves):

$$\frac{72888 + 89033x - 94314x^2 + 39280x^3}{161304(8 + 24x + 8x^2 - 15x^3 + 8x^4)} + \frac{\text{RootSum}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \frac{74897 \text{Log}[x-\#1] - 57489 \text{Log}[x-\#1] \#1 + 19640 \text{Log}[x-\#1] \#1^2}{24+16\#1-45\#1^2+32\#1^3} \&\right]}{80652}$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{(a + bx)^6}{6b}$$

Result (type 1, 61 leaves):

$$a^5 x + \frac{5}{2} a^4 b x^2 + \frac{10}{3} a^3 b^2 x^3 + \frac{5}{2} a^2 b^3 x^4 + a b^4 x^5 + \frac{b^5 x^6}{6}$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - (c + d x)^2} dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$\frac{\text{ArcTanh}[c + d x]}{d}$$

Result (type 3, 32 leaves):

$$-\frac{\text{Log}[1 - c - d x]}{2 d} + \frac{\text{Log}[1 + c + d x]}{2 d}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - (1 + x)^2} dx$$

Optimal (type 3, 4 leaves, 2 steps):

$$\text{ArcTanh}[1 + x]$$

Result (type 3, 15 leaves):

$$-\frac{\text{Log}[x]}{2} + \frac{1}{2} \text{Log}[2 + x]$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{x^3}{a + b (c + d x)^3} dx$$

Optimal (type 3, 234 leaves, 11 steps):

$$\frac{x}{b d^3} + \frac{(a - 3 a^{1/3} b^{2/3} c^2 + b c^3) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{4/3} d^4} - \frac{(a + 3 a^{1/3} b^{2/3} c^2 + b c^3) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{4/3} d^4} +$$

$$\frac{(a + 3 a^{1/3} b^{2/3} c^2 + b c^3) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{4/3} d^4} - \frac{c \operatorname{Log}\left[a + b (c + d x)^3\right]}{b d^4}$$

Result (type 7, 132 leaves):

$$-\frac{1}{3 b^2 d^4} \left(-3 b d x + \operatorname{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{a \operatorname{Log}[x - \#1] + b c^3 \operatorname{Log}[x - \#1] + 3 b c^2 d \operatorname{Log}[x - \#1] \#1 + 3 b c d^2 \operatorname{Log}[x - \#1] \#1^2}{c^2 + 2 c d \#1 + d^2 \#1^2} \& \right] \right)$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{x^2}{a + b (c + d x)^3} dx$$

Optimal (type 3, 210 leaves, 9 steps):

$$\frac{c (2 a^{1/3} - b^{1/3} c) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{2/3} d^3} + \frac{c (2 a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{2/3} d^3} -$$

$$\frac{c (2 a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{2/3} d^3} + \frac{\operatorname{Log}\left[a + b (c + d x)^3\right]}{3 b d^3}$$

Result (type 7, 81 leaves):

$$\frac{\operatorname{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{\operatorname{Log}[x - \#1] \#1^2}{c^2 + 2 c d \#1 + d^2 \#1^2} \& \right]}{3 b d}$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{x}{a + b (c + d x)^3} dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$-\frac{(a^{1/3} - b^{1/3} c) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{2/3} d^2} - \frac{(a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{2/3} d^2} + \frac{(a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{2/3} d^2}$$

Result (type 7, 79 leaves):

$$\frac{\text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{\text{Log}[x - \#1] \#1}{c^2 + 2 c d \#1 + d^2 \#1^2} \&\right]}{3 b d}$$

Problem 107: Result is not expressed in closed-form.

$$\int \frac{1}{x (a + b (c + d x)^3)} dx$$

Optimal (type 3, 224 leaves, 11 steps):

$$\frac{b^{1/3} c \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a^{2/3} - a^{1/3} b^{1/3} c + b^{2/3} c^2)} + \frac{\text{Log}[x]}{a + b c^3} - \frac{\text{Log}[a^{1/3} + b^{1/3} (c + d x)]}{3 a^{2/3} (a^{1/3} + b^{1/3} c)} - \frac{(2 a^{1/3} - b^{1/3} c) \text{Log}[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2]}{6 a^{2/3} (a^{2/3} - a^{1/3} b^{1/3} c + b^{2/3} c^2)}$$

Result (type 7, 119 leaves):

$$-\frac{1}{3 (a + b c^3)} \left(-3 \text{Log}[x] + \text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{3 c^2 \text{Log}[x - \#1] + 3 c d \text{Log}[x - \#1] \#1 + d^2 \text{Log}[x - \#1] \#1^2}{c^2 + 2 c d \#1 + d^2 \#1^2} \&\right] \right)$$

Problem 108: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (a + b (c + d x)^3)} dx$$

Optimal (type 3, 314 leaves, 11 steps):

$$-\frac{1}{(a + b c^3) x} + \frac{b^{1/3} (a^{1/3} - b^{1/3} c) (a^{1/3} + b^{1/3} c)^3 d \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a + b c^3)^2} - \frac{3 b c^2 d \text{Log}[x]}{(a + b c^3)^2} + \frac{b^{1/3} (a^{1/3} (a - 2 b c^3) - b^{1/3} c (2 a - b c^3)) d \text{Log}[a^{1/3} + b^{1/3} (c + d x)]}{3 a^{2/3} (a + b c^3)^2} - \frac{b^{1/3} (a^{1/3} (a - 2 b c^3) - b^{1/3} c (2 a - b c^3)) d \text{Log}[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2]}{6 a^{2/3} (a + b c^3)^2} + \frac{b c^2 d \text{Log}[a + b (c + d x)^3]}{(a + b c^3)^2}$$

Result (type 7, 173 leaves):

$$\frac{1}{3 (a + b c^3)^2 x} \left(-3 (a + b c^3 + 3 b c^2 d x \text{Log}[x]) + d x \text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{1}{c^2 + 2 c d \#1 + d^2 \#1^2} (-3 a c \text{Log}[x - \#1] + 6 b c^4 \text{Log}[x - \#1] - a d \text{Log}[x - \#1] \#1 + 8 b c^3 d \text{Log}[x - \#1] \#1 + 3 b c^2 d^2 \text{Log}[x - \#1] \#1^2) \&\right] \right)$$

Problem 109: Result is not expressed in closed-form.

$$\int \frac{1}{x^3 (a + b (c + d x)^3)} dx$$

Optimal (type 3, 393 leaves, 11 steps):

$$\begin{aligned} & -\frac{1}{2 (a + b c^3) x^2} + \frac{3 b c^2 d}{(a + b c^3)^2 x} + \frac{b^{2/3} (a^{1/3} + b^{1/3} c)^3 (a - 3 a^{2/3} b^{1/3} c + b c^3) d^2 \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a + b c^3)^3} - \\ & \frac{3 b c (a - 2 b c^3) d^2 \operatorname{Log}[x]}{(a + b c^3)^3} - \frac{b^{2/3} (a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6) d^2 \operatorname{Log}[a^{1/3} + b^{1/3} (c + d x)]}{3 a^{2/3} (a + b c^3)^3} + \\ & \frac{b^{2/3} (a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6) d^2 \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2]}{6 a^{2/3} (a + b c^3)^3} + \frac{b c (a - 2 b c^3) d^2 \operatorname{Log}[a + b (c + d x)^3]}{(a + b c^3)^3} \end{aligned}$$

Result (type 7, 244 leaves):

$$\begin{aligned} & -\frac{1}{6 (a + b c^3)^3 x^2} \left(3 (a + b c^3) (a + b c^2 (c - 6 d x)) + 18 b c (a - 2 b c^3) d^2 x^2 \operatorname{Log}[x] + \right. \\ & \quad \left. 2 d^2 x^2 \operatorname{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{1}{c^2 + 2 c d \#1 + d^2 \#1^2} (a^2 \operatorname{Log}[x - \#1] - 16 a b c^3 \operatorname{Log}[x - \#1] + \right. \right. \\ & \quad \left. \left. 10 b^2 c^6 \operatorname{Log}[x - \#1] - 12 a b c^2 d \operatorname{Log}[x - \#1] \#1 + 15 b^2 c^5 d \operatorname{Log}[x - \#1] \#1 - 3 a b c d^2 \operatorname{Log}[x - \#1] \#1^2 + 6 b^2 c^4 d^2 \operatorname{Log}[x - \#1] \#1^2) \& \right] \right) \end{aligned}$$

Problem 110: Result is not expressed in closed-form.

$$\int \frac{x^3}{a + b (c + d x)^4} dx$$

Optimal (type 3, 356 leaves, 16 steps):

$$\begin{aligned} & \frac{3 c^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} (c + d x)^2}{\sqrt{a}}\right]}{2 \sqrt{a} \sqrt{b} d^4} + \frac{c (3 \sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c + d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \\ & \frac{c (3 \sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c + d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \frac{c (3 \sqrt{a} - \sqrt{b} c^2) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \\ & \frac{c (3 \sqrt{a} - \sqrt{b} c^2) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{\operatorname{Log}[a + b (c + d x)^4]}{4 b d^4} \end{aligned}$$

Result (type 7, 106 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1^3}{c^3+3 c^2 d \#1+3 c d^2 \#1^2+d^3 \#1^3} \&\right]}{4 b d}$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{x^2}{a + b (c + d x)^4} dx$$

Optimal (type 3, 318 leaves, 14 steps):

$$\begin{aligned} & - \frac{c \text{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b} d^3} - \frac{(\sqrt{a} + \sqrt{b} c^2) \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3} + \frac{(\sqrt{a} + \sqrt{b} c^2) \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3} + \\ & \frac{(\sqrt{a} - \sqrt{b} c^2) \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3} - \frac{(\sqrt{a} - \sqrt{b} c^2) \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3} \end{aligned}$$

Result (type 7, 106 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1^2}{c^3+3 c^2 d \#1+3 c d^2 \#1^2+d^3 \#1^3} \&\right]}{4 b d}$$

Problem 112: Result is not expressed in closed-form.

$$\int \frac{x}{a + b (c + d x)^4} dx$$

Optimal (type 3, 261 leaves, 14 steps):

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{2 \sqrt{a} \sqrt{b} d^2} + \frac{c \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{1/4} d^2} - \frac{c \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{1/4} d^2} + \\ & \frac{c \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{1/4} d^2} - \frac{c \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{1/4} d^2} \end{aligned}$$

Result (type 7, 104 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1}{c^3+3 c^2 d \#1+3 c d^2 \#1^2+d^3 \#1^3} \&\right]}{4 b d}$$

Problem 114: Result is not expressed in closed-form.

$$\int \frac{1}{x (a + b (c + d x)^4)} dx$$

Optimal (type 3, 393 leaves, 18 steps):

$$\begin{aligned} & - \frac{\sqrt{b} c^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{2 \sqrt{a} (a + b c^4)} + \frac{b^{1/4} c (\sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a + b c^4)} - \\ & \frac{b^{1/4} c (\sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a + b c^4)} + \frac{\operatorname{Log}[x]}{a + b c^4} - \frac{b^{1/4} c (\sqrt{a} - \sqrt{b} c^2) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2\right]}{4 \sqrt{2} a^{3/4} (a + b c^4)} + \\ & \frac{b^{1/4} c (\sqrt{a} - \sqrt{b} c^2) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2\right]}{4 \sqrt{2} a^{3/4} (a + b c^4)} - \frac{\operatorname{Log}[a + b (c + d x)^4]}{4 (a + b c^4)} \end{aligned}$$

Result (type 7, 163 leaves):

$$\begin{aligned} & - \frac{1}{4 (a + b c^4)} \left(-4 \operatorname{Log}[x] + \operatorname{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \right. \right. \\ & \left. \left. \frac{4 c^3 \operatorname{Log}[x - \#1] + 6 c^2 d \operatorname{Log}[x - \#1] \#1 + 4 c d^2 \operatorname{Log}[x - \#1] \#1^2 + d^3 \operatorname{Log}[x - \#1] \#1^3}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \& \right] \right) \end{aligned}$$

Problem 115: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (a + b (c + d x)^4)} dx$$

Optimal (type 3, 496 leaves, 18 steps):

$$\begin{aligned}
& - \frac{1}{(a + b c^4) x} - \frac{\sqrt{b} c (a - b c^4) d \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{\sqrt{a} (a + b c^4)^2} + \frac{b^{1/4} \left(\sqrt{a} (a - 3 b c^4) + \sqrt{b} c^2 (3 a - b c^4)\right) d \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a + b c^4)^2} \\
& - \frac{b^{1/4} \left(\sqrt{a} (a - 3 b c^4) + \sqrt{b} c^2 (3 a - b c^4)\right) d \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a + b c^4)^2} - \frac{4 b c^3 d \operatorname{Log}[x]}{(a + b c^4)^2} \\
& + \frac{b^{1/4} \left(\sqrt{a} (a - 3 b c^4) - \sqrt{b} c^2 (3 a - b c^4)\right) d \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c + dx) + \sqrt{b} (c + dx)^2\right]}{4 \sqrt{2} a^{3/4} (a + b c^4)^2} \\
& + \frac{b^{1/4} \left(\sqrt{a} (a - 3 b c^4) - \sqrt{b} c^2 (3 a - b c^4)\right) d \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c + dx) + \sqrt{b} (c + dx)^2\right]}{4 \sqrt{2} a^{3/4} (a + b c^4)^2} + \frac{b c^3 d \operatorname{Log}[a + b (c + dx)^4]}{(a + b c^4)^2}
\end{aligned}$$

Result (type 7, 238 leaves):

$$\begin{aligned}
& \frac{1}{4 (a + b c^4)^2 x} \left(-4 (a + b c^4 + 4 b c^3 d x \operatorname{Log}[x]) + \right. \\
& \left. d x \operatorname{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \left(-6 a c^2 \operatorname{Log}[x - \#1] + 10 b c^6 \operatorname{Log}[x - \#1] - 4 a c d \operatorname{Log}[x - \#1] \#1 + \right. \right. \right. \\
& \left. \left. \left. 20 b c^5 d \operatorname{Log}[x - \#1] \#1 - a d^2 \operatorname{Log}[x - \#1] \#1^2 + 15 b c^4 d^2 \operatorname{Log}[x - \#1] \#1^2 + 4 b c^3 d^3 \operatorname{Log}[x - \#1] \#1^3 \right) / (c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3) \& \right] \right)
\end{aligned}$$

Problem 120: Result is not expressed in closed-form.

$$\int \frac{1}{a + 8 x - 8 x^2 + 4 x^3 - x^4} dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$- \frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1-\sqrt{4+a}}} + \frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 57 leaves):

$$- \frac{1}{4} \operatorname{RootSum}\left[a + 8 \#1 - 8 \#1^2 + 4 \#1^3 - \#1^4 \&, \frac{\operatorname{Log}[x - \#1]}{-2 + 4 \#1 - 3 \#1^2 + \#1^3} \& \right]$$

Problem 121: Result is not expressed in closed-form.

$$\int \frac{1}{(a + 8 x - 8 x^2 + 4 x^3 - x^4)^2} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{(10+3a+\sqrt{4+a}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} + \frac{(10+3a-\sqrt{4+a}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 150 leaves):

$$\frac{(-1+x)(6+a-2x+x^2)}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\operatorname{RootSum}\left[a+8\sqrt[3]{-8+4\sqrt[3]{1^3-1^4}} \&, \frac{12 \operatorname{Log}[x-\sqrt[3]{1}]+3a \operatorname{Log}[x-\sqrt[3]{1}]-2 \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1+\operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1^2}}}{-2+4\sqrt[3]{1}-3\sqrt[3]{1^2+\sqrt[3]{1^3}}}\right] \&}{16(12+7a+a^2)}$$

Problem 122: Result is not expressed in closed-form.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal (type 3, 252 leaves, 6 steps):

$$\frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2(3+a-2(-1+x)^2-(-1+x)^4)} -$$

$$\frac{3(80+7a^2+14\sqrt{4+a}+a(47+4\sqrt{4+a})) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} - \frac{3(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 254 leaves):

$$\frac{1}{128} \left(\frac{16(-1+x)(6+a-2x+x^2)}{(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))^2} + \frac{4(-1+x)(7a^2+6(32-14x+7x^2)+a(79-24x+12x^2))}{(3+a)^2(4+a)^2(a-x(-8+8x-4x^2+x^3))} - \right.$$

$$\left. \frac{1}{(12+7a+a^2)^2} \operatorname{RootSum}\left[a+8\sqrt[3]{-8+4\sqrt[3]{1^3-1^4}} \&, \frac{1}{-2+4\sqrt[3]{1}-3\sqrt[3]{1^2+\sqrt[3]{1^3}}}\right] \& \right)$$

$$\left(108 \operatorname{Log}[x-\sqrt[3]{1}] + 55a \operatorname{Log}[x-\sqrt[3]{1}] + 7a^2 \operatorname{Log}[x-\sqrt[3]{1}] - 28 \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1} - 8a \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1} + 14 \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1^2} + 4a \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1^2} \right) \& \right)$$

Problem 127: Result is not expressed in closed-form.

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} + \frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2\sqrt{4+a}}$$

Result (type 7, 59 leaves):

$$-\frac{1}{4} \text{RootSum}\left[a + 8\#1 - 8\#1^2 + 4\#1^3 - \#1^4 \&, \frac{\text{Log}[x - \#1] \#1}{-2 + 4\#1 - 3\#1^2 + \#1^3} \&\right]$$

Problem 128: Result is not expressed in closed-form.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx$$

Optimal (type 3, 231 leaves, 10 steps):

$$\frac{1 + (-1+x)^2}{4(4+a)(3+a-2(-1+x)^2 - (-1+x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2 - (-1+x)^4)} -$$

$$\frac{(10+3a+\sqrt{4+a})\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} + \frac{(10+3a-\sqrt{4+a})\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{4(4+a)^{3/2}}$$

Result (type 7, 166 leaves):

$$\frac{a + 2x - ax + ax^2 + x^3}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\text{RootSum}\left[a + 8\#1 - 8\#1^2 + 4\#1^3 - \#1^4 \&, \frac{6\text{Log}[x-\#1]+a\text{Log}[x-\#1]+4\text{Log}[x-\#1]\#1+2a\text{Log}[x-\#1]\#1+\text{Log}[x-\#1]\#1^2}{-2+4\#1-3\#1^2+\#1^3} \&\right]}{16(12+7a+a^2)}$$

Problem 129: Result is not expressed in closed-form.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx$$

Optimal (type 3, 349 leaves, 12 steps):

$$\frac{1 + (-1+x)^2}{8(4+a)(3+a-2(-1+x)^2 - (-1+x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(-1+x)^2 - (-1+x)^4)} +$$

$$\frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2 - (-1+x)^4)^2} + \frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2(3+a-2(-1+x)^2 - (-1+x)^4)} -$$

$$\frac{3(80+7a^2+14\sqrt{4+a}+a(47+4\sqrt{4+a}))\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} - \frac{3(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}})\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}} + \frac{3\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{16(4+a)^{5/2}}$$

Result (type 7, 284 leaves):

$$\frac{1}{128} \left(\frac{16(a+2x-ax+ax^2+x^3)}{(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))^2} + \frac{4(a^2(5-5x+6x^2)+6(-14+28x-12x^2+7x^3)+a(-7+31x+12x^3))}{(3+a)^2(4+a)^2(a-x(-8+8x-4x^2+x^3))} - \right.$$

$$\left. \frac{1}{(12+7a+a^2)^2} \text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4 \&, \frac{1}{-2+4\#1-3\#1^2+\#1^3} (72\text{Log}[x-\#1]+31a\text{Log}[x-\#1]+ \right.$$

$$\left. 3a^2\text{Log}[x-\#1]+8\text{Log}[x-\#1]\#1+16a\text{Log}[x-\#1]\#1+4a^2\text{Log}[x-\#1]\#1+14\text{Log}[x-\#1]\#1^2+4a\text{Log}[x-\#1]\#1^2) \& \right]$$

Problem 134: Result is not expressed in closed-form.

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal (type 3, 99 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2\sqrt{1-\sqrt{4+a}}} - \frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2\sqrt{1+\sqrt{4+a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{\sqrt{4+a}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{4} \text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4 \&, \frac{\text{Log}[x-\#1]\#1^2}{-2+4\#1-3\#1^2+\#1^3} \& \right]$$

Problem 135: Result is not expressed in closed-form.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal (type 3, 225 leaves, 11 steps):

$$\frac{1 + (-1+x)^2}{2(4+a)(3+a-2(-1+x)^2 - (-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2 - (-1+x)^4)} -$$

$$\frac{(4+a+\sqrt{4+a}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8(3+a)(4+a)\sqrt{1-\sqrt{4+a}}} - \frac{(4+a-\sqrt{4+a}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8(3+a)(4+a)\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2(4+a)^{3/2}}$$

Result (type 7, 182 leaves):

$$\frac{2x(4-3x+2x^2) + a(1+x-x^2+x^3)}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\operatorname{RootSum}\left[a+8\sqrt[3]{1}-8\sqrt[3]{1^2}+4\sqrt[3]{1^3}-\sqrt[3]{1^4}\ \&, \frac{-a \operatorname{Log}[x-\sqrt[3]{1}]+4 \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1+2a \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1+4 \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1+a \operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1^2}}}}{-2+4\sqrt[3]{1}-3\sqrt[3]{1^2}+\sqrt[3]{1^3}}\ \& \right]}{16(12+7a+a^2)}$$

Problem 136: Result is not expressed in closed-form.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal (type 3, 545 leaves, 14 steps):

$$- \frac{(-1)^{1/3} \left(2(-1)^{1/3}b + 3a^{1/3}c^{2/3} \right) \operatorname{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}} \right]}{3\sqrt{3} \left(1 + (-1)^{1/3} \right)^2 a^{5/6} b^2 \sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}} c^{2/3}} -$$

$$\frac{(2b-3a^{1/3}c^{2/3}) \operatorname{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}} \right]}{9\sqrt{3} a^{5/6} b^2 \sqrt{4b-3a^{1/3}c^{2/3}} c^{2/3}} - \frac{(-1)^{2/3} \left(2b + 3(-1)^{1/3}a^{1/3}c^{2/3} \right) \operatorname{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}} \right]}{3\sqrt{3} \left(1 - (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/6} b^2 \sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}} c^{2/3}} -$$

$$\frac{\operatorname{Log}\left[3a + 3a^{2/3}c^{1/3}x + bx^2 \right]}{18a^{2/3}b^2c^{1/3}} + \frac{\operatorname{Log}\left[3a - 3(-1)^{1/3}a^{2/3}c^{1/3}x + bx^2 \right]}{6 \left(1 + (-1)^{1/3} \right)^2 a^{2/3}b^2c^{1/3}} + \frac{(-1)^{1/3} \operatorname{Log}\left[3a + 3(-1)^{2/3}a^{2/3}c^{1/3}x + bx^2 \right]}{18a^{2/3}b^2c^{1/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3} \operatorname{RootSum}\left[27a^3 + 27a^2b\sqrt[3]{1} + 27a^2c\sqrt[3]{1^3} + 9ab^2\sqrt[3]{1^4} + b^3\sqrt[3]{1^6}\ \&, \frac{\operatorname{Log}[x-\sqrt[3]{1}]\sqrt[3]{1^3}}{18a^2b + 27a^2c\sqrt[3]{1} + 12ab^2\sqrt[3]{1^2} + 2b^3\sqrt[3]{1^4}}\ \& \right]$$

Problem 137: Result is not expressed in closed-form.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal (type 3, 487 leaves, 14 steps):

$$\frac{\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{3\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{7/6}b\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{1/3}} - \frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}a^{7/6}b\sqrt{4b-3a^{1/3}c^{2/3}}c^{1/3}} + \frac{(-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{3\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{7/6}b\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{1/3}} + \frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{18\left(1+(-1)^{1/3}\right)^2a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&, \frac{\text{Log}\left[x-\#1\right]\#1^2}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Problem 138: Result is not expressed in closed-form.

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6} dx$$

Optimal (type 3, 334 leaves, 8 steps):

$$\frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{11/6}b\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{2/3}} + \frac{2\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}a^{11/6}b\sqrt{4b-3a^{1/3}c^{2/3}}c^{2/3}} + \frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{11/6}b\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{2/3}}$$

Result (type 7, 97 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&, \frac{\text{Log}\left[x-\#1\right]\#1}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Problem 139: Result is not expressed in closed-form.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6} dx$$

Optimal (type 3, 469 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{13/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{1/3}} - \frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}a^{13/6}\sqrt{4b-3a^{1/3}c^{2/3}}c^{1/3}} +$$

$$\frac{(-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{13/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{1/3}} - \frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{162a^{7/3}c^{2/3}} +$$

$$\frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{54\left(1+(-1)^{1/3}\right)^2a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{162a^{7/3}c^{2/3}}$$

Result (type 7, 95 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\ \&, \frac{\text{Log}[x-\#1]}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\ \&\right]$$

Problem 140: Result is not expressed in closed-form.

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6} dx$$

Optimal (type 3, 522 leaves, 14 steps):

$$\frac{(-1)^{1/3}\left(2(-1)^{1/3}b+3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{2/3}} -$$

$$\frac{(2b-3a^{1/3}c^{2/3})\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{81\sqrt{3}a^{17/6}\sqrt{4b-3a^{1/3}c^{2/3}}c^{2/3}} - \frac{(2(-1)^{2/3}b-3a^{1/3}c^{2/3})\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{17/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{2/3}} +$$

$$\frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{162a^{8/3}c^{1/3}} - \frac{\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{54\left(1+(-1)^{1/3}\right)^2a^{8/3}c^{1/3}} - \frac{(-1)^{1/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{162a^{8/3}c^{1/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\ \&, \frac{\text{Log}[x-\#1]}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\ \&\right]$$

Problem 141: Result is not expressed in closed-form.

$$\int \frac{1}{x (27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6)} dx$$

Optimal (type 3, 563 leaves, 14 steps):

$$\frac{(b - (-1)^{2/3} a^{1/3} c^{2/3}) \operatorname{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3} - 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1 + (-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}} c^{1/3}} + \frac{(b - a^{1/3} c^{2/3}) \operatorname{ArcTan}\left[\frac{3 a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 a^{1/3} c^{2/3}}}\right]}{27 \sqrt{3} a^{19/6} \sqrt{4 b - 3 a^{1/3} c^{2/3}} c^{1/3}} +$$

$$\frac{(-1)^{2/3} \left((-1)^{2/3} b - a^{1/3} c^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1 - (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}} c^{1/3}} + \frac{\operatorname{Log}[x]}{27 a^3} - \frac{\left(3 a^{1/3} - \frac{b}{c^{2/3}}\right) \operatorname{Log}\left[3 a + 3 a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{10/3}} -$$

$$\frac{\left(b + i \sqrt{3} b + 6 a^{1/3} c^{2/3}\right) \operatorname{Log}\left[3 a - 3 (-1)^{1/3} a^{2/3} c^{1/3} x + b x^2\right]}{972 a^{10/3} c^{2/3}} - \frac{\left(3 a^{1/3} - \frac{(-1)^{2/3} b}{c^{2/3}}\right) \operatorname{Log}\left[3 a + 3 (-1)^{2/3} a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{10/3}}$$

Result (type 7, 157 leaves):

$$-\frac{1}{81 a^3} \left(-3 \operatorname{Log}[x] + \operatorname{RootSum}\left[27 a^3 + 27 a^2 b \#1^2 + 27 a^2 c \#1^3 + 9 a b^2 \#1^4 + b^3 \#1^6 \&, \right. \right. \\ \left. \left. \frac{27 a^2 b \operatorname{Log}[x - \#1] + 27 a^2 c \operatorname{Log}[x - \#1] \#1 + 9 a b^2 \operatorname{Log}[x - \#1] \#1^2 + b^3 \operatorname{Log}[x - \#1] \#1^4}{18 a^2 b + 27 a^2 c \#1 + 12 a b^2 \#1^2 + 2 b^3 \#1^4} \& \right] \right)$$

Problem 142: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6)} dx$$

Optimal (type 3, 645 leaves, 14 steps):

$$\begin{aligned}
& -\frac{1}{27 a^3 x} + \frac{\left(2 (-1)^{2/3} b^2 + 12 (-1)^{1/3} a^{1/3} b c^{2/3} + 9 a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3} - 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{81 \sqrt{3} \left(1 + (-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}} c^{2/3}} + \\
& \frac{\left(2 b^2 - 12 a^{1/3} b c^{2/3} + 9 a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 a^{1/3} c^{2/3}}}\right]}{243 \sqrt{3} a^{23/6} \sqrt{4 b - 3 a^{1/3} c^{2/3}} c^{2/3}} + \\
& \frac{(-1)^{2/3} \left(2 b^2 + 12 (-1)^{1/3} a^{1/3} b c^{2/3} + 9 (-1)^{2/3} a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{81 \sqrt{3} \left(1 - (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}} c^{2/3}} - \frac{(2 b - 3 a^{1/3} c^{2/3}) \operatorname{Log}\left[3 a + 3 a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{11/3} c^{1/3}} + \\
& \frac{(2 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}) \operatorname{Log}\left[3 a - 3 (-1)^{1/3} a^{2/3} c^{1/3} x + b x^2\right]}{162 \left(1 + (-1)^{1/3}\right)^2 a^{11/3} c^{1/3}} + \frac{(-1)^{1/3} (2 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}) \operatorname{Log}\left[3 a + 3 (-1)^{2/3} a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{11/3} c^{1/3}}
\end{aligned}$$

Result (type 7, 163 leaves):

$$-\frac{1}{81 a^3 x} \left(3 + x \operatorname{RootSum}\left[27 a^3 + 27 a^2 b \#1^2 + 27 a^2 c \#1^3 + 9 a b^2 \#1^4 + b^3 \#1^6 \&, \frac{27 a^2 b \operatorname{Log}[x - \#1] + 27 a^2 c \operatorname{Log}[x - \#1] \#1 + 9 a b^2 \operatorname{Log}[x - \#1] \#1^2 + b^3 \operatorname{Log}[x - \#1] \#1^4}{18 a^2 b \#1 + 27 a^2 c \#1^2 + 12 a b^2 \#1^3 + 2 b^3 \#1^5} \&]\right)$$

Problem 143: Result is not expressed in closed-form.

$$\int \frac{x^5}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 395 leaves, 14 steps):

$$\begin{aligned}
& -\frac{(-2)^{1/3} \left(1 + (-2)^{1/3} 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}}\right]}{3^{5/6} \sqrt{8 + 9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\left(\frac{3}{2}\right)^{1/6} \left(1 - (-3)^{2/3} 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4 - 3 (-3)^{2/3} 2^{1/3})}}\right]}{\left(1 + (-1)^{1/3}\right)^2 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(1 - 2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{1}{216} \left(36 + 2^{2/3} \times 3^{1/3} \left(1 + i \sqrt{3}\right)\right) \operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right] + \\
& \frac{1}{108} \left(18 - (-2)^{2/3} 3^{1/3}\right) \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right] + \frac{1}{108} \left(18 - 2^{2/3} \times 3^{1/3}\right) \operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]
\end{aligned}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^4}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 144: Result is not expressed in closed-form.

$$\int \frac{x^4}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\frac{(-1)^{2/3} \left(3 (-3)^{2/3} - 2^{2/3}\right) \text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right] + \left(9 - (-2)^{2/3} 3^{1/3}\right) \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{9 \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{2(4-3(-3)^{2/3} 2^{1/3})} + 27 \sqrt{3(8+9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3})}} -$$

$$\frac{\left(9 - 2^{2/3} \times 3^{1/3}\right) \text{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{27 \sqrt{6(-4+3 \times 2^{1/3} \times 3^{2/3})}} + \frac{\text{Log}[6-3(-3)^{1/3} 2^{2/3} x + x^2]}{6 \times 2^{2/3} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^2} + \frac{\left(-\frac{1}{3}\right)^{1/3} \text{Log}[6+3(-2)^{2/3} 3^{1/3} x + x^2]}{18 \times 2^{2/3}} - \frac{\text{Log}[6+3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{18 \times 2^{2/3} \times 3^{1/3}}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^3}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 145: Result is not expressed in closed-form.

$$\int \frac{x^3}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{6 \times 2^{1/6} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \frac{(-1)^{1/3} \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{9 \times 2^{2/3} \times 3^{5/6} \sqrt{8+9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{18 \times 2^{1/6} \times 3^{5/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}} -$$

$$\frac{(-1)^{2/3} \text{Log}[6-3(-3)^{1/3} 2^{2/3} x + x^2]}{36 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} + \frac{(-1)^{2/3} \text{Log}[6+3(-2)^{2/3} 3^{1/3} x + x^2]}{108 \times 2^{1/3} \times 3^{2/3}} + \frac{\text{Log}[6+3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{108 \times 2^{1/3} \times 3^{2/3}}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^2}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 146: Result is not expressed in closed-form.

$$\int \frac{x^2}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 248 leaves, 8 steps):

$$\frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{27 \times 2^{5/6} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{81 \times 2^{1/3} \times 3^{1/6} \sqrt{8+9 \#1^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6}(3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{81 \times 2^{5/6} \times 3^{1/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}}$$

Result (type 7, 59 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 147: Result is not expressed in closed-form.

$$\int \frac{x}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$\begin{aligned} & - \frac{\text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{36 \times 2^{1/6} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \frac{(-1)^{1/3} \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{54 \times 2^{2/3} \times 3^{5/6} \sqrt{8+9 \#1^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\text{ArcTanh}\left[\frac{2^{1/6}(3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{108 \times 2^{1/6} \times 3^{5/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}} + \\ & \frac{(-1)^{2/3} \text{Log}\left[6-3(-3)^{1/3} 2^{2/3} x + x^2\right]}{216 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} - \frac{(-1)^{2/3} \text{Log}\left[6+3(-2)^{2/3} 3^{1/3} x + x^2\right]}{648 \times 2^{1/3} \times 3^{2/3}} - \frac{\text{Log}\left[6+3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{648 \times 2^{1/3} \times 3^{2/3}} \end{aligned}$$

Result (type 7, 57 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1]}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 148: Result is not expressed in closed-form.

$$\int \frac{1}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\frac{(-1)^{2/3} \left(3 (-3)^{2/3} - 2^{2/3} \right) \operatorname{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}} \right] + \left(9 - (-2)^{2/3} 3^{1/3} \right) \operatorname{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}} \right]}{324 \times 3^{1/6} \left(1 + (-1)^{1/3} \right)^2 \sqrt{2 (4-3 (-3)^{2/3} 2^{1/3})} + 972 \sqrt{3 (8+9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3})}} \\ - \frac{(9 - 2^{2/3} \times 3^{1/3}) \operatorname{ArcTanh} \left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{972 \sqrt{6 (-4+3 \times 2^{1/3} \times 3^{2/3})}} - \frac{\operatorname{Log} [6-3 (-3)^{1/3} 2^{2/3} x + x^2]}{216 \times 2^{2/3} \times 3^{1/3} \left(1 + (-1)^{1/3} \right)^2} - \frac{\left(-\frac{1}{3} \right)^{1/3} \operatorname{Log} [6+3 (-2)^{2/3} 3^{1/3} x + x^2]}{648 \times 2^{2/3}} + \frac{\operatorname{Log} [6+3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{648 \times 2^{2/3} \times 3^{1/3}}$$

Result (type 7, 62 leaves):

$$\frac{1}{6} \operatorname{RootSum} \left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\operatorname{Log} [x - \#1]}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]$$

Problem 149: Result is not expressed in closed-form.

$$\int \frac{1}{x (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} dx$$

Optimal (type 3, 415 leaves, 14 steps):

$$\frac{(-1)^{2/3} \left((-2)^{2/3} - 2 \times 3^{2/3} \right) \operatorname{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}} \right] + (-1)^{2/3} \left((-3)^{1/3} + 3 \times 2^{1/3} \right) \operatorname{ArcTan} \left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}} \right]}{216 \times 2^{1/3} \times 3^{5/6} \sqrt{8+9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}} - 216 \times 6^{1/6} \left(1 + (-1)^{1/3} \right)^2 \sqrt{4-3 (-3)^{2/3} 2^{1/3}}}$$

$$+ \frac{(1 - 2^{1/3} \times 3^{2/3}) \operatorname{ArcTanh} \left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{216 \times 2^{1/6} \times 3^{5/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\operatorname{Log} [x]}{216} - \frac{(36 + 2^{2/3} \times 3^{1/3} (1 + i \sqrt{3})) \operatorname{Log} [6-3 (-3)^{1/3} 2^{2/3} x + x^2]}{46656}$$

$$- \frac{(18 - (-2)^{2/3} 3^{1/3}) \operatorname{Log} [6+3 (-2)^{2/3} 3^{1/3} x + x^2]}{23328} - \frac{(18 - 2^{2/3} \times 3^{1/3}) \operatorname{Log} [6+3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{23328}$$

Result (type 7, 103 leaves):

$$\frac{\text{Log}[x]}{216} - \frac{\text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \text{Log}[x-\#1] + 324 \text{Log}[x-\#1] \#1 + 18 \text{Log}[x-\#1] \#1^2 + \text{Log}[x-\#1] \#1^4}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]}{1296}$$

Problem 150: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} dx$$

Optimal (type 3, 448 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{216 x} - \frac{\left(27 (-6)^{1/3} - (-2)^{2/3} + 12 \times 3^{2/3}\right) \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{5832 \times 3^{1/6} \sqrt{8+9 \#1^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{(-1)^{2/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3} - 2^{1/3}\right) \text{ArcTan}\left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{1944 \times 6^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3 (-3)^{2/3} 2^{1/3}}} \\ & \frac{\left(2^{1/3} + 27 \times 3^{1/3} - 6 \times 6^{2/3}\right) \text{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{5832 \times 6^{1/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}} - \frac{(-1)^{2/3} \left(9 + (-3)^{1/3} 2^{2/3}\right) \text{Log}\left[6-3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{1296 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} + \\ & \frac{\left(3 (-6)^{2/3} + 2 (-2)^{1/3}\right) \text{Log}\left[6+3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{7776 \times 3^{1/3}} - \frac{\left(2^{2/3} - 3 \times 3^{2/3}\right) \text{Log}\left[6+3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{3888 \times 6^{1/3}} \end{aligned}$$

Result (type 7, 109 leaves):

$$\frac{1}{216 x} - \frac{\text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \text{Log}[x-\#1] + 324 \text{Log}[x-\#1] \#1 + 18 \text{Log}[x-\#1] \#1^2 + \text{Log}[x-\#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&\right]}{1296}$$

Problem 151: Result is not expressed in closed-form.

$$\int \frac{x^8}{(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)^2} dx$$

Optimal (type 3, 1064 leaves, 23 steps):

$$\begin{aligned}
& - \frac{\left(-\frac{1}{3}\right)^{1/3} \left(9 \left(6 + (-3)^{1/3} 2^{2/3}\right) + \left(2 - 2^{2/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3}\right)\right) x\right)}{162 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right)} - \\
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(9 \left(6 - (-2)^{2/3} 3^{1/3}\right) + \left(2 + 27 (-2)^{2/3} 3^{1/3} + 12 (-2)^{1/3} 3^{2/3}\right) x\right)}{729 \times 2^{2/3} \left(8 + 9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right)} + \frac{9 \left(6 - 2^{2/3} \times 3^{1/3}\right) + \left(2 + 2^{2/3} \left(27 \times 3^{1/3} - 6 \times 6^{2/3}\right)\right) x}{1458 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} - \\
& \frac{i \left((-2)^{2/3} + 6 \times 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)}}\right] - (-1)^{1/3} \left(2 + 27 (-2)^{2/3} 3^{1/3} + 12 (-2)^{1/3} 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)}}\right]}{162 \times 2^{5/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}} - 162 \times 2^{1/6} \times 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)^{3/2}} - \\
& \frac{(-1)^{1/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3} - 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right)}}\right] - \left(i 2^{2/3} - 9 \times 3^{1/6} - 3 i 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right)}}\right]}{81 \sqrt{2} 3^{5/6} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right)^{3/2} + 162 \times 2^{5/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(1 + 3 \times 2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3}\right)}}\right] - \left(2^{1/3} + 27 \times 3^{1/3} - 6 \times 6^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3}\right)}}\right]}{1458 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}} + 81 \sqrt{2} 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \\
& \frac{\operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{972 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^4} + \frac{i \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{972 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^5} - \frac{\operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{8748 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-7884 + 324 x - 3990 x^2 - 11610 x^3 - 203 x^4 - 9 x^5}{34182 \left(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6\right)} - \frac{1}{205092} \\
& \operatorname{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6, \frac{324 \operatorname{Log}[x - \#1] - 96 \operatorname{Log}[x - \#1] \#1 + 324 \operatorname{Log}[x - \#1] \#1^2 + 406 \operatorname{Log}[x - \#1] \#1^3 + 9 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 152: Result is not expressed in closed-form.

$$\int \frac{x^7}{\left(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6\right)^2} dx$$

Optimal (type 3, 1005 leaves, 23 steps):

$$\begin{aligned}
& - \frac{2 \left((-1)^{1/3} 3^{2/3} + 9 \times 6^{1/3} \right) - 9 \left((-2)^{2/3} + 2 (-1)^{1/3} 3^{2/3} \right) x}{972 \times 2^{2/3} \left(1 + (-1)^{1/3} \right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right)} - \\
& \frac{(-6)^{1/3} \left(9 (-2)^{1/3} + 2 \times 3^{1/3} \right) - 9 \left(1 + (-2)^{1/3} 3^{2/3} \right) x}{4374 \left(8 + 9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right)} + \frac{2 \left(2 - 3 \times 2^{1/3} \times 3^{2/3} \right) - 3 \left(6 - 2^{2/3} \times 3^{1/3} \right) x}{2916 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right)} + \\
& \frac{\left(9 i + 3^{1/3} \left(2 i 2^{2/3} - 9 \times 3^{1/6} + 2 \times 2^{2/3} \sqrt{3} \right) \right) \text{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2 x}{\sqrt{6 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)}} \right]}{5832 \left(1 + (-1)^{1/3} \right)^5 \sqrt{2 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)}} + \frac{\left(1 + (-2)^{1/3} 3^{2/3} \right) \text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)}} \right]}{54 \sqrt{6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)^{3/2}} + \\
& \frac{\left(9 \times 3^{1/6} + i \left(4 \times 2^{2/3} - 3 \times 3^{2/3} \right) \right) \text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)}} \right]}{1944 \times 3^{2/3} \left(1 + (-1)^{1/3} \right)^5 \sqrt{2 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)}} - \frac{(-1)^{1/3} \left((-3)^{1/3} + 3 \times 2^{1/3} \right) \text{ArcTan} \left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x \right)}{\sqrt{3 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)}} \right]}{54 \sqrt{2} 3^{5/6} \left(1 + (-1)^{1/3} \right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)^{3/2}} + \\
& \frac{\left(1 - 2^{1/3} \times 3^{2/3} \right) \text{ArcTanh} \left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x \right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3} \right)}} \right]}{54 \sqrt{6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} + \frac{\left(2 \times 2^{2/3} + 3 \times 3^{2/3} \right) \text{ArcTanh} \left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x \right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3} \right)}} \right]}{26244 \times 3^{1/6} \sqrt{2 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3} \right)}} + \\
& \frac{i \text{Log} \left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right]}{648 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^5} - \frac{\left(i + \sqrt{3} \right) \text{Log} \left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right]}{1296 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^5} - \frac{\text{Log} \left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right]}{17496 \times 2^{2/3} \times 3^{1/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\frac{648 - 96 x + 432 x^2 + 908 x^3 - 18 x^4 + 73 x^5}{68364 \left(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6 \right)} + \frac{1}{410184}$$

$$\text{RootSum} \left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{96 \text{Log} [x - \#1] - 216 \text{Log} [x - \#1] \#1 + 96 \text{Log} [x - \#1] \#1^2 - 36 \text{Log} [x - \#1] \#1^3 + 73 \text{Log} [x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]$$

Problem 153: Result is not expressed in closed-form.

$$\int \frac{x^6}{\left(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6 \right)^2} dx$$

Optimal (type 3, 677 leaves, 14 steps):

$$\begin{aligned}
& \frac{9(-2)^{2/3} + 6^{1/3}(9 + (-3)^{1/3}2^{2/3})x}{2916 \times 2^{2/3}(1 + (-1)^{1/3})^4(4 - 3(-3)^{2/3}2^{1/3})(6 - 3(-3)^{1/3}2^{2/3}x + x^2)} + \frac{9 \times 2^{2/3} + (-1)^{1/3}3^{2/3}(2 + 3(-2)^{1/3}3^{2/3})x}{13122 \times 2^{2/3}(8 + 9i2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3})(6 + 3(-2)^{2/3}3^{1/3}x + x^2)} + \\
& \frac{3 \times 2^{2/3} \times 3^{1/3} - (2 - 3 \times 2^{1/3} \times 3^{2/3})x}{8748 \times 2^{2/3} \times 3^{1/3}(4 - 3 \times 2^{1/3} \times 3^{2/3})(6 + 3 \times 2^{2/3} \times 3^{1/3}x + x^2)} + \frac{(-1)^{1/3}(3(-3)^{2/3} - 2^{2/3}) \operatorname{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3}2^{1/3})}}\right]}{486 \times 6^{5/6}(1 + (-1)^{1/3})^4(4 - 3(-3)^{2/3}2^{1/3})^{3/2}} + \\
& \frac{(3(-3)^{2/3} + (-1)^{1/3}2^{2/3}) \operatorname{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3}3^{2/3})}}\right]}{486 \times 6^{5/6}(1 - (-1)^{1/3})^2(1 + (-1)^{1/3})^4(4 + 3(-2)^{1/3}3^{2/3})^{3/2}} - \frac{(2^{2/3} - 3 \times 3^{2/3}) \operatorname{ArcTanh}\left[\frac{2^{1/6}(3 \cdot 3^{1/3} + 2^{1/3}x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{486 \times 6^{5/6}(1 - (-1)^{1/3})^2(1 + (-1)^{1/3})^4(-4 + 3 \times 2^{1/3} \times 3^{2/3})^{3/2}} + \\
& \frac{\left(-\frac{1}{3}\right)^{1/6} \operatorname{Log}[6 - 3(-3)^{1/3}2^{2/3}x + x^2]}{5832 \times 2^{1/3}(1 + (-1)^{1/3})^5} - \frac{i \operatorname{Log}[6 + 3(-2)^{2/3}3^{1/3}x + x^2]}{5832 \times 2^{1/3} \times 3^{1/6}(1 + (-1)^{1/3})^5} + \frac{\operatorname{Log}[6 + 3 \times 2^{2/3} \times 3^{1/3}x + x^2]}{52488 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\frac{-96 + 108x - 64x^2 - 72x^3 + 73x^4 - 3x^5}{68364(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} - \frac{1}{410184}$$

$$\operatorname{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \operatorname{Log}[x - \#1] - 32 \operatorname{Log}[x - \#1] \#1 + 108 \operatorname{Log}[x - \#1] \#1^2 - 146 \operatorname{Log}[x - \#1] \#1^3 + 3 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&\right]$$

Problem 154: Result is not expressed in closed-form.

$$\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 682 leaves, 17 steps):

$$\begin{aligned}
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(4 - (-3)^{1/3} 2^{2/3} x\right)}{1944 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3(-3)^{2/3} 2^{1/3}\right) \left(6 - 3(-3)^{1/3} 2^{2/3} x + x^2\right)} + \frac{\left(-\frac{1}{3}\right)^{1/3} \left(4 + (-2)^{2/3} 3^{1/3} x\right)}{8748 \times 2^{2/3} \left(8 + 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3(-2)^{2/3} 3^{1/3} x + x^2\right)} - \\
& \frac{4 + 2^{2/3} \times 3^{1/3} x}{17496 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} - \frac{\text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{4374 \times 2^{5/6} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^4 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \\
& \frac{\text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{4374 \sqrt{3} \left(8 - 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{i \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{1458 \times 2^{5/6} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4+3(-2)^{1/3} 3^{2/3}}} - \\
& \frac{\text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{4374 \sqrt{3} \left(8 + 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{8748 \sqrt{6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{39366 \times 2^{5/6} \times 3^{1/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{972 - 144x + 648x^2 + 729x^3 - 27x^4 + 4x^5}{615276(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} + \frac{1}{3691656} \text{RootSum}\left[216 + 108\#1^2 + 324\#1^3 + 18\#1^4 + \#1^6 \&, \right. \\
& \left. \frac{144 \text{Log}[x - \#1] - 324 \text{Log}[x - \#1] \#1 + 2043 \text{Log}[x - \#1] \#1^2 - 54 \text{Log}[x - \#1] \#1^3 + 4 \text{Log}[x - \#1] \#1^4}{36\#1 + 162\#1^2 + 12\#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 155: Result is not expressed in closed-form.

$$\int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 850 leaves, 23 steps):

$$\begin{aligned}
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(3(-3)^{1/3} 2^{2/3} - 2x\right)}{5832 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3(-3)^{2/3} 2^{1/3}\right) \left(6 - 3(-3)^{1/3} 2^{2/3} x + x^2\right)} - \frac{\left(-\frac{1}{3}\right)^{1/3} \left(3(-2)^{2/3} 3^{1/3} + 2x\right)}{26244 \times 2^{2/3} \left(8 + 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3(-2)^{2/3} 3^{1/3} x + x^2\right)} - \\
& \frac{3 \times 3^{1/3} + 2^{1/3} x}{52488 \left(9 \times 2^{1/3} - 4 \times 3^{1/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} + \frac{(-1)^{1/3} \operatorname{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{729 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^4 \left(8 - 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \\
& \frac{(-1)^{1/3} \operatorname{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{2916 \times 2^{1/6} \times 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(4 + 3(-2)^{1/3} 3^{2/3}\right)^{3/2}} - \frac{(i + \sqrt{3}) \operatorname{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{11664 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 + 3(-2)^{1/3} 3^{2/3}}} - \\
& \frac{i \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3(-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3(4-3(-3)^{2/3} 2^{1/3})}}\right]}{5832 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 - 3(-3)^{2/3} 2^{1/3}}} + \frac{\operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{26244 \times 2^{1/6} \times 3^{5/6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3(-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}}\right]}{52488 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} - \\
& \frac{\operatorname{Log}\left[6 - 3(-3)^{1/3} 2^{2/3} x + x^2\right]}{34992 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^4} + \frac{i \operatorname{Log}\left[6 + 3(-2)^{2/3} 3^{1/3} x + x^2\right]}{34992 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^5} - \frac{\operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{314928 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-288 + 324x - 1458x^2 - 216x^3 + 8x^4 - 9x^5}{1230552(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} - \frac{1}{7383312} \operatorname{RootSum}\left[216 + 108\#1^2 + 324\#1^3 + 18\#1^4 + \#1^6 \&, \right. \\
& \left. \frac{324 \operatorname{Log}[x - \#1] - 2628 \operatorname{Log}[x - \#1] \#1 + 324 \operatorname{Log}[x - \#1] \#1^2 - 16 \operatorname{Log}[x - \#1] \#1^3 + 9 \operatorname{Log}[x - \#1] \#1^4}{36\#1 + 162\#1^2 + 12\#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 156: Result is not expressed in closed-form.

$$\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 873 leaves, 23 steps):

$$\begin{aligned}
& \frac{(-6)^{1/3} \left(2 (-3)^{1/3} + 9 \times 2^{1/3} \right) - 3x}{157464 \left(8 - 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right)} - \frac{(-6)^{1/3} \left(9 (-2)^{1/3} + 2 \times 3^{1/3} \right) + 3x}{157464 \left(8 + 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right)} - \\
& \frac{2 \times 2^{1/3} - 3 \times 6^{2/3} - 3^{1/3} x}{104976 \left(9 \times 2^{1/3} - 4 \times 3^{1/3} \right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right)} + \frac{\text{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}} \right]}{26244 \sqrt{3} \left(8 - 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} - \\
& \frac{\left(9i - 3^{1/3} \left(2i 2^{2/3} + 9 \times 3^{1/6} + 2 \times 2^{2/3} \sqrt{3} \right) \right) \text{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}} \right]}{209952 \left(1 + (-1)^{1/3} \right)^5 \sqrt{2 (4-3 (-3)^{2/3} 2^{1/3})}} - \frac{\text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}} \right]}{26244 \sqrt{3} \left(8 + 9i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} + \\
& \frac{\left(9i + 3^{1/3} \left(4i 2^{2/3} - 9 \times 3^{1/6} \right) \right) \text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}} \right]}{209952 \left(1 + (-1)^{1/3} \right)^5 \sqrt{2 (4+3 (-2)^{1/3} 3^{2/3})}} - \frac{\text{ArcTanh} \left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{52488 \sqrt{6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} + \frac{\left(2 \times 2^{2/3} - 3 \times 3^{2/3} \right) \text{ArcTanh} \left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{944784 \times 3^{1/6} \sqrt{2 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}} - \\
& \frac{i \text{Log} \left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right]}{23328 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^5} + \frac{\left(i + \sqrt{3} \right) \text{Log} \left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right]}{46656 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^5} + \frac{\text{Log} \left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right]}{629856 \times 2^{2/3} \times 3^{1/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{972 - 3942x + 648x^2 + 96x^3 - 27x^4 + 4x^5}{3691656 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} + \frac{1}{11074968} \\
& \text{RootSum} \left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{1971 \text{Log} [x - \#1] - 162 \text{Log} [x - \#1] \#1 + 72 \text{Log} [x - \#1] \#1^2 - 27 \text{Log} [x - \#1] \#1^3 + 2 \text{Log} [x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 157: Result is not expressed in closed-form.

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 986 leaves, 23 steps):

$$\begin{aligned}
& - \frac{27 \left((-2)^{2/3} + 2 (-1)^{1/3} 3^{2/3} \right) - 6^{1/3} \left(9 + (-3)^{1/3} 2^{2/3} \right) x}{104976 \times 2^{2/3} \left(1 + (-1)^{1/3} \right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right)} - \\
& \frac{27 \times 2^{2/3} \left(1 + (-2)^{1/3} 3^{2/3} \right) - (-1)^{1/3} 3^{2/3} \left(2 + 3 (-2)^{1/3} 3^{2/3} \right) x}{472392 \times 2^{2/3} \left(8 + 9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right)} + \frac{9 \left(6 - 2^{2/3} \times 3^{1/3} \right) - \left(2 - 3 \times 2^{1/3} \times 3^{2/3} \right) x}{314928 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right)} - \\
& \frac{\left(1 + i \sqrt{3} + 3 \times 2^{1/3} \times 3^{2/3} \right) \text{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)}} \right]}{8748 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^4 \left(8 - 9 i 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} + \frac{\left(3 (-3)^{2/3} + (-1)^{1/3} 2^{2/3} \right) \text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)}} \right]}{17496 \times 6^{5/6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)^{3/2}} + \\
& \frac{\left(i + \sqrt{3} \right) \text{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)}} \right]}{34992 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3} \right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}}} + \frac{i \text{ArcTan} \left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x \right)}{\sqrt{3 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right)}} \right]}{17496 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3} \right)^5 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(2^{2/3} - 3 \times 3^{2/3} \right) \text{ArcTanh} \left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x \right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3} \right)}} \right]}{17496 \times 6^{5/6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} - \frac{\text{ArcTanh} \left[\frac{2^{1/6} \left(3 \cdot 3^{1/3} + 2^{1/3} x \right)}{\sqrt{3 \left(-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3} \right)}} \right]}{157464 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \\
& \frac{\left(i + \sqrt{3} \right) \text{Log} \left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right]}{419904 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3} \right)^5} - \frac{i \text{Log} \left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right]}{209952 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3} \right)^5} + \frac{\text{Log} \left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right]}{1889568 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-7884 + 324 x - 2724 x^2 - 216 x^3 + 8 x^4 - 9 x^5}{7383312 \left(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6 \right)} - \frac{1}{44299872} \text{RootSum} \left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \right. \\
& \left. \frac{324 \text{Log} [x - \#1] + 2436 \text{Log} [x - \#1] \#1 + 324 \text{Log} [x - \#1] \#1^2 - 16 \text{Log} [x - \#1] \#1^3 + 9 \text{Log} [x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int (b + 2 c x) (b x + c x^2)^{13} dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{14} (b x + c x^2)^{14}$$

Result (type 1, 172 leaves):

$$\frac{b^{14} x^{14}}{14} + b^{13} c x^{15} + \frac{13}{2} b^{12} c^2 x^{16} + 26 b^{11} c^3 x^{17} + \frac{143}{2} b^{10} c^4 x^{18} + 143 b^9 c^5 x^{19} + \frac{429}{2} b^8 c^6 x^{20} +$$

$$\frac{1716}{7} b^7 c^7 x^{21} + \frac{429}{2} b^6 c^8 x^{22} + 143 b^5 c^9 x^{23} + \frac{143}{2} b^4 c^{10} x^{24} + 26 b^3 c^{11} x^{25} + \frac{13}{2} b^2 c^{12} x^{26} + b c^{13} x^{27} + \frac{c^{14} x^{28}}{14}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int x^{14} (b + 2 c x^2) (b x + c x^3)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{28} x^{28} (b + c x^2)^{14}$$

Result (type 1, 182 leaves):

$$\frac{b^{14} x^{28}}{28} + \frac{1}{2} b^{13} c x^{30} + \frac{13}{4} b^{12} c^2 x^{32} + 13 b^{11} c^3 x^{34} + \frac{143}{4} b^{10} c^4 x^{36} + \frac{143}{2} b^9 c^5 x^{38} + \frac{429}{4} b^8 c^6 x^{40} +$$

$$\frac{858}{7} b^7 c^7 x^{42} + \frac{429}{4} b^6 c^8 x^{44} + \frac{143}{2} b^5 c^9 x^{46} + \frac{143}{4} b^4 c^{10} x^{48} + 13 b^3 c^{11} x^{50} + \frac{13}{4} b^2 c^{12} x^{52} + \frac{1}{2} b c^{13} x^{54} + \frac{c^{14} x^{56}}{28}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int x^{28} (b + 2 c x^3) (b x + c x^4)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{42} x^{42} (b + c x^3)^{14}$$

Result (type 1, 186 leaves):

$$\frac{b^{14} x^{42}}{42} + \frac{1}{3} b^{13} c x^{45} + \frac{13}{6} b^{12} c^2 x^{48} + \frac{26}{3} b^{11} c^3 x^{51} + \frac{143}{6} b^{10} c^4 x^{54} + \frac{143}{3} b^9 c^5 x^{57} + \frac{143}{2} b^8 c^6 x^{60} +$$

$$\frac{572}{7} b^7 c^7 x^{63} + \frac{143}{2} b^6 c^8 x^{66} + \frac{143}{3} b^5 c^9 x^{69} + \frac{143}{6} b^4 c^{10} x^{72} + \frac{26}{3} b^3 c^{11} x^{75} + \frac{13}{6} b^2 c^{12} x^{78} + \frac{1}{3} b c^{13} x^{81} + \frac{c^{14} x^{84}}{42}$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-7(-1+n)} (b + 2c x^n)}{(b x + c x^{1+n})^8} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$-\frac{x^{-7n}}{7n(b + c x^n)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7b^{14}n(b + c x^n)^7} x^{-7n} (b^{14} + 1716b^7c^7x^{7n} + 12012b^6c^8x^{8n} + 36036b^5c^9x^{9n} + 60060b^4c^{10}x^{10n} + 60060b^3c^{11}x^{11n} + 36036b^2c^{12}x^{12n} + 12012bc^{13}x^{13n} + 1716c^{14}x^{14n})$$

Problem 192: Result more than twice size of optimal antiderivative.

$$\int (b + 2cx + 3dx^2)(a + bx + cx^2 + dx^3)^7 dx$$

Optimal (type 1, 21 leaves, 1 step):

$$\frac{1}{8}(a + bx + cx^2 + dx^3)^8$$

Result (type 1, 143 leaves):

$$\frac{1}{8}x(b + x(c + dx)) \left(8a^7 + 28a^6x(b + x(c + dx)) + 56a^5x^2(b + x(c + dx))^2 + 70a^4x^3(b + x(c + dx))^3 + 56a^3x^4(b + x(c + dx))^4 + 28a^2x^5(b + x(c + dx))^5 + 8ax^6(b + x(c + dx))^6 + x^7(b + x(c + dx))^7 \right)$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int (b + 3dx^2)(a + bx + dx^3)^7 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{8}(a + bx + dx^3)^8$$

Result (type 1, 127 leaves):

$$\frac{1}{8} x (b + d x^2) \left(8 a^7 + 28 a^6 x (b + d x^2) + 56 a^5 x^2 (b + d x^2)^2 + 70 a^4 x^3 (b + d x^2)^3 + 56 a^3 x^4 (b + d x^2)^4 + 28 a^2 x^5 (b + d x^2)^5 + 8 a x^6 (b + d x^2)^6 + x^7 (b + d x^2)^7 \right)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int (b + 3 d x^2) (b x + d x^3)^7 dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{8} (b x + d x^3)^8$$

Result (type 1, 98 leaves):

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7 b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Problem 197: Result more than twice size of optimal antiderivative.

$$\int x^7 (b + d x^2)^7 (b + 3 d x^2) dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{8} x^8 (b + d x^2)^8$$

Result (type 1, 98 leaves):

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7 b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int (2 c x + 3 d x^2) (a + c x^2 + d x^3)^7 dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8} (a + c x^2 + d x^3)^8$$

Result (type 1, 115 leaves):

$$\frac{1}{8} x^2 (c + d x) \left(8 a^7 + 28 a^6 x^2 (c + d x) + 56 a^5 x^4 (c + d x)^2 + 70 a^4 x^6 (c + d x)^3 + 56 a^3 x^8 (c + d x)^4 + 28 a^2 x^{10} (c + d x)^5 + 8 a x^{12} (c + d x)^6 + x^{14} (c + d x)^7 \right)$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int (2 c x + 3 d x^2) (c x^2 + d x^3)^7 dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{1}{8} (c x^2 + d x^3)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int x^7 (c x + d x^2)^7 (2 c x + 3 d x^2) dx$$

Optimal (type 1, 14 leaves, 2 steps):

$$\frac{1}{8} x^{16} (c + d x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 201: Result more than twice size of optimal antiderivative.

$$\int x^{14} (c + d x)^7 (2 c x + 3 d x^2) dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{8} x^{16} (c + d x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int x (2c + 3dx) (a + cx^2 + dx^3)^7 dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Result (type 1, 115 leaves):

$$\frac{1}{8} x^2 (c + dx) \left(8a^7 + 28a^6x^2(c + dx) + 56a^5x^4(c + dx)^2 + 70a^4x^6(c + dx)^3 + 56a^3x^8(c + dx)^4 + 28a^2x^{10}(c + dx)^5 + 8ax^{12}(c + dx)^6 + x^{14}(c + dx)^7 \right)$$

Problem 203: Result more than twice size of optimal antiderivative.

$$\int x (2c + 3dx) (cx^2 + dx^3)^7 dx$$

Optimal (type 1, 14 leaves, 2 steps):

$$\frac{1}{8} x^{16} (c + dx)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int x^8 (2c + 3dx) (cx + dx^2)^7 dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8} x^8 (cx + dx^2)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int x^{15} (c + d x)^7 (2 c + 3 d x) dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{8} x^{16} (c + d x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int (a + b x) \left(1 + \left(a x + \frac{b x^2}{2} \right)^4 \right) dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{1}{160} x^5 (2 a + b x)^5$$

Result (type 1, 80 leaves):

$$a x + \frac{b x^2}{2} + \frac{a^5 x^5}{5} + \frac{1}{2} a^4 b x^6 + \frac{1}{2} a^3 b^2 x^7 + \frac{1}{4} a^2 b^3 x^8 + \frac{1}{16} a b^4 x^9 + \frac{b^5 x^{10}}{160}$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int (a + b x) \left(1 + \left(c + a x + \frac{b x^2}{2} \right)^4 \right) dx$$

Optimal (type 1, 31 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{1}{5} \left(c + a x + \frac{b x^2}{2} \right)^5$$

Result (type 1, 108 leaves):

$$\frac{1}{160} x (2 a + b x) \left(80 + 80 c^4 + 16 a^4 x^4 + 32 a^3 b x^5 + 24 a^2 b^2 x^6 + 8 a b^3 x^7 + b^4 x^8 + 80 c^3 x (2 a + b x) + 40 c^2 x^2 (2 a + b x)^2 + 10 c x^3 (2 a + b x)^3 \right)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int (a + b x) \left(1 + \left(c + a x + \frac{b x^2}{2} \right)^n \right) dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{\left(c + a x + \frac{b x^2}{2} \right)^{1+n}}{1+n}$$

Result (type 3, 73 leaves):

$$\frac{2 c \left(c + a x + \frac{b x^2}{2} \right)^n + 2 a x \left(1 + n + \left(c + a x + \frac{b x^2}{2} \right)^n \right) + b x^2 \left(1 + n + \left(c + a x + \frac{b x^2}{2} \right)^n \right)}{2 (1+n)}$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int (a + c x^2) \left(1 + \left(a x + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 30 leaves, 2 steps):

$$a x + \frac{c x^3}{3} + \frac{1}{6} \left(a x + \frac{c x^3}{3} \right)^6$$

Result (type 1, 93 leaves):

$$a x + \frac{c x^3}{3} + \frac{a^6 x^6}{6} + \frac{1}{3} a^5 c x^8 + \frac{5}{18} a^4 c^2 x^{10} + \frac{10}{81} a^3 c^3 x^{12} + \frac{5}{162} a^2 c^4 x^{14} + \frac{1}{243} a c^5 x^{16} + \frac{c^6 x^{18}}{4374}$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int (a + c x^2) \left(1 + \left(d + a x + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 31 leaves, 2 steps):

$$a x + \frac{c x^3}{3} + \frac{1}{6} \left(d + a x + \frac{c x^3}{3} \right)^6$$

Result (type 1, 140 leaves):

$$\frac{1}{4374}x (3a + cx^2) \left(1458 + 1458d^5 + 243a^5x^5 + 405a^4cx^7 + 270a^3c^2x^9 + 90a^2c^3x^{11} + 15a^4c^4x^{13} + c^5x^{15} + 1215d^4(3ax + cx^3) + 540d^3(3ax + cx^3)^2 + 135d^2(3ax + cx^3)^3 + 18d(3ax + cx^3)^4 \right)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 34 leaves, 2 steps):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{x^{12}(3b + 2cx)^6}{279936}$$

Result (type 1, 98 leaves):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{b^6x^{12}}{384} + \frac{1}{96}b^5cx^{13} + \frac{5}{288}b^4c^2x^{14} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{486}bc^5x^{17} + \frac{c^6x^{18}}{4374}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6$$

Result (type 1, 146 leaves):

$$\frac{1}{279936}x^2(3b + 2cx) \left(46656 + 46656d^5 + 243b^5x^{10} + 810b^4cx^{11} + 1080b^3c^2x^{12} + 720b^2c^3x^{13} + 240b^4c^4x^{14} + 32c^5x^{15} + 19440d^4x^2(3b + 2cx) + 4320d^3x^4(3b + 2cx)^2 + 540d^2x^6(3b + 2cx)^3 + 36dx^8(3b + 2cx)^4 \right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 46 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^6$$

Result (type 1, 244 leaves):

$$\begin{aligned} & \frac{a^6 x^6}{6} + \frac{1}{6} a^5 x^7 (3 b + 2 c x) + \frac{5}{72} a^4 x^8 (3 b + 2 c x)^2 + \frac{5}{324} a^3 x^9 (3 b + 2 c x)^3 + \\ & \frac{5 a^2 x^{10} (3 b + 2 c x)^4}{2592} + a \left(x + \frac{b^5 x^{11}}{32} + \frac{5}{48} b^4 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^3 x^{14} + \frac{5}{162} b c^4 x^{15} + \frac{c^5 x^{16}}{243} \right) + \frac{1}{279936} \\ & x^2 (729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b (243 + c^5 x^{15}) + 64 c x (1458 + c^5 x^{15})) \end{aligned}$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int (a + b x + c x^2) \left(1 + \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 47 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^6$$

Result (type 1, 248 leaves):

$$\begin{aligned} & \frac{1}{279936} x (6 a + x (3 b + 2 c x)) \left(46656 + 46656 d^5 + 7776 a^5 x^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + \right. \\ & 240 b c^4 x^{14} + 32 c^5 x^{15} + 6480 a^4 x^6 (3 b + 2 c x) + 2160 a^3 x^7 (3 b + 2 c x)^2 + 360 a^2 x^8 (3 b + 2 c x)^3 + 30 a x^9 (3 b + 2 c x)^4 + \\ & \left. 19440 d^4 x (6 a + x (3 b + 2 c x)) + 4320 d^3 x^2 (6 a + x (3 b + 2 c x))^2 + 540 d^2 x^3 (6 a + x (3 b + 2 c x))^3 + 36 d x^4 (6 a + x (3 b + 2 c x))^4 \right) \end{aligned}$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int (1 + 2 x) (x + x^2)^3 (-18 + 7 (x + x^2)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 x^4 (1 + x)^4 - 36 x^7 (1 + x)^7 + \frac{49}{10} x^{10} (1 + x)^{10}$$

Result (type 1, 96 leaves):

$$\begin{aligned} & 81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \\ & \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10} \end{aligned}$$

Problem 222: Result more than twice size of optimal antiderivative.

$$\int x^3 (1+x)^3 (1+2x) (-18+7x^3(1+x)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

Result (type 1, 96 leaves):

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Problem 227: Result is not expressed in closed-form.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{a + bx + cx^2 + dx^3 + ax^4} dx$$

Optimal (type 3, 605 leaves, 9 steps):

$$\left(\left(4 a^2 B + b \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right) D - a \left(A \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right) + b C - \sqrt{8 a^2 + b^2 - 4 a c} C + 2 c D \right) \right) \right. \\
\left. \text{ArcTan} \left[\frac{b - \sqrt{8 a^2 + b^2 - 4 a c} + 4 a x}{\sqrt{2} \sqrt{4 a^2 + 2 a c - b \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right)}} \right] \right) / \left(\sqrt{2} a \sqrt{8 a^2 + b^2 - 4 a c} \sqrt{4 a^2 + 2 a c - b \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right)} \right) - \\
\left(\left(4 a^2 B + b \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right) D - a \left(A \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right) + b C + \sqrt{8 a^2 + b^2 - 4 a c} C + 2 c D \right) \right) \right. \\
\left. \text{ArcTan} \left[\frac{b + \sqrt{8 a^2 + b^2 - 4 a c} + 4 a x}{\sqrt{2} \sqrt{4 a^2 + 2 a c - b \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right)}} \right] \right) / \left(\sqrt{2} a \sqrt{8 a^2 + b^2 - 4 a c} \sqrt{4 a^2 + 2 a c - b \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right)} \right) - \\
\frac{\left(2 a \left(A - C \right) + \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right) D \right) \text{Log} \left[2 a + \left(b - \sqrt{8 a^2 + b^2 - 4 a c} \right) x + 2 a x^2 \right]}{4 a \sqrt{8 a^2 + b^2 - 4 a c}} + \\
\frac{\left(2 a \left(A - C \right) + \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right) D \right) \text{Log} \left[2 a + \left(b + \sqrt{8 a^2 + b^2 - 4 a c} \right) x + 2 a x^2 \right]}{4 a \sqrt{8 a^2 + b^2 - 4 a c}}$$

Result (type 7, 98 leaves):

$$\text{RootSum} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, \frac{A \text{Log} [x - \#1] + B \text{Log} [x - \#1] \#1 + C \text{Log} [x - \#1] \#1^2 + D \text{Log} [x - \#1] \#1^3}{b + 2 c \#1 + 3 b \#1^2 + 4 a \#1^3} \& \right]$$

Problem 250: Result is not expressed in closed-form.

$$\int \frac{x^3 (5 + x + 3 x^2 + 2 x^3)}{2 + x + 5 x^2 + x^3 + 2 x^4} dx$$

Optimal (type 3, 307 leaves, 13 steps):

$$\begin{aligned}
& -\frac{1}{28} (35 - 9i\sqrt{7})x - \frac{1}{28} (35 + 9i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2 + \frac{1}{42} (7 - 5i\sqrt{7})x^3 + \\
& \frac{1}{42} (7 + 5i\sqrt{7})x^3 + \frac{11(9i + 5\sqrt{7}) \operatorname{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right] - 11(9i - 5\sqrt{7}) \operatorname{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right]}{4\sqrt{14(35+i\sqrt{7})} - 4\sqrt{14(35-i\sqrt{7})}} + \\
& \frac{3}{112} (7 - 11i\sqrt{7}) \operatorname{Log}[4 + (1 - i\sqrt{7})x + 4x^2] + \frac{3}{112} (7 + 11i\sqrt{7}) \operatorname{Log}[4 + (1 + i\sqrt{7})x + 4x^2]
\end{aligned}$$

Result (type 7, 109 leaves):

$$\frac{1}{6} \left(x(-15 + 3x + 2x^2) + 3 \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{10 \operatorname{Log}[x - \#1] + \operatorname{Log}[x - \#1] \#1 + 19 \operatorname{Log}[x - \#1] \#1^2 + 3 \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \& \right] \right)$$

Problem 251: Result is not expressed in closed-form.

$$\int \frac{x^2(5 + x + 3x^2 + 2x^3)}{2 + x + 5x^2 + x^3 + 2x^4} dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$\begin{aligned}
& \frac{1}{14} (7 - 5i\sqrt{7})x + \frac{1}{14} (7 + 5i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2 - \frac{(53i + \sqrt{7}) \operatorname{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right]}{2\sqrt{14(35+i\sqrt{7})}} + \\
& \frac{(53i - \sqrt{7}) \operatorname{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right]}{2\sqrt{14(35-i\sqrt{7})}} - \frac{1}{56} (35 + 9i\sqrt{7}) \operatorname{Log}[4 + (1 - i\sqrt{7})x + 4x^2] - \frac{1}{56} (35 - 9i\sqrt{7}) \operatorname{Log}[4 + (1 + i\sqrt{7})x + 4x^2]
\end{aligned}$$

Result (type 7, 101 leaves):

$$x + \frac{x^2}{2} - \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{2 \operatorname{Log}[x - \#1] + 3 \operatorname{Log}[x - \#1] \#1 + \operatorname{Log}[x - \#1] \#1^2 + 5 \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \& \right]$$

Problem 252: Result is not expressed in closed-form.

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal (type 3, 230 leaves, 11 steps):

$$\frac{1}{14} (7 - 5i\sqrt{7}) x + \frac{1}{14} (7 + 5i\sqrt{7}) x - \frac{(19i + 7\sqrt{7}) \operatorname{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right]}{\sqrt{14(35+i\sqrt{7})}} +$$

$$\frac{(19i - 7\sqrt{7}) \operatorname{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right]}{\sqrt{14(35-i\sqrt{7})}} + \frac{1}{28} (7 + 5i\sqrt{7}) \operatorname{Log}[4 + (1 - i\sqrt{7})x + 4x^2] + \frac{1}{28} (7 - 5i\sqrt{7}) \operatorname{Log}[4 + (1 + i\sqrt{7})x + 4x^2]$$

Result (type 7, 94 leaves):

$$x + 2 \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{-\operatorname{Log}[x - \#1] + 2 \operatorname{Log}[x - \#1] \#1 - 2 \operatorname{Log}[x - \#1] \#1^2 + \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 253: Result is not expressed in closed-form.

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$$

Optimal (type 3, 198 leaves, 9 steps):

$$\frac{(19i + 7\sqrt{7}) \operatorname{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right]}{\sqrt{14(35+i\sqrt{7})}} - \frac{(19i - 7\sqrt{7}) \operatorname{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right]}{\sqrt{14(35-i\sqrt{7})}} +$$

$$\frac{1}{28} (7 + 5i\sqrt{7}) \operatorname{Log}[4 + (1 - i\sqrt{7})x + 4x^2] + \frac{1}{28} (7 - 5i\sqrt{7}) \operatorname{Log}[4 + (1 + i\sqrt{7})x + 4x^2]$$

Result (type 7, 90 leaves):

$$\operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{5 \operatorname{Log}[x - \#1] + \operatorname{Log}[x - \#1] \#1 + 3 \operatorname{Log}[x - \#1] \#1^2 + 2 \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 254: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 245 leaves, 13 steps):

$$\begin{aligned} & \frac{(53 + i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i - \sqrt{7} + 8ix}{2(35 - i\sqrt{7})}\right]}{2\sqrt{14(35 - i\sqrt{7})}} + \frac{(53 - i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i + \sqrt{7} + 8ix}{2(35 + i\sqrt{7})}\right]}{2\sqrt{14(35 + i\sqrt{7})}} + \frac{1}{28}(35 - 9i\sqrt{7}) \operatorname{Log}[x] + \\ & \frac{1}{28}(35 + 9i\sqrt{7}) \operatorname{Log}[x] - \frac{1}{56}(35 - 9i\sqrt{7}) \operatorname{Log}[4i + (i - \sqrt{7})x + 4ix^2] - \frac{1}{56}(35 + 9i\sqrt{7}) \operatorname{Log}[4i + (i + \sqrt{7})x + 4ix^2] \end{aligned}$$

Result (type 7, 101 leaves):

$$\frac{5 \operatorname{Log}[x]}{2} - \frac{1}{2} \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{3 \operatorname{Log}[x - \#1] + 19 \operatorname{Log}[x - \#1] \#1 + \operatorname{Log}[x - \#1] \#1^2 + 10 \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 255: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 281 leaves, 13 steps):

$$\begin{aligned} & -\frac{35 - 9i\sqrt{7}}{28x} - \frac{35 + 9i\sqrt{7}}{28x} + \frac{11(9 + 5i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i - \sqrt{7} + 8ix}{2(35 - i\sqrt{7})}\right]}{4\sqrt{14(35 - i\sqrt{7})}} - \frac{11(9 - 5i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i + \sqrt{7} + 8ix}{2(35 + i\sqrt{7})}\right]}{4\sqrt{14(35 + i\sqrt{7})}} - \frac{3}{56}(7 - 11i\sqrt{7}) \operatorname{Log}[x] - \\ & \frac{3}{56}(7 + 11i\sqrt{7}) \operatorname{Log}[x] + \frac{3}{112}(7 + 11i\sqrt{7}) \operatorname{Log}[4i + (i - \sqrt{7})x + 4ix^2] + \frac{3}{112}(7 - 11i\sqrt{7}) \operatorname{Log}[4i + (i + \sqrt{7})x + 4ix^2] \end{aligned}$$

Result (type 7, 109 leaves):

$$-\frac{5}{2x} - \frac{3 \operatorname{Log}[x]}{4} + \frac{1}{4} \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{-35 \operatorname{Log}[x - \#1] + 13 \operatorname{Log}[x - \#1] \#1 - 17 \operatorname{Log}[x - \#1] \#1^2 + 6 \operatorname{Log}[x - \#1] \#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 256: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 317 leaves, 13 steps):

$$\begin{aligned} & -\frac{35 - 9i\sqrt{7}}{56x^2} - \frac{35 + 9i\sqrt{7}}{56x^2} + \frac{3(7 - 11i\sqrt{7})}{56x} + \frac{3(7 + 11i\sqrt{7})}{56x} + \frac{(355 - 73i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i - \sqrt{7} + 8ix}{\sqrt{2(35 - i\sqrt{7})}}\right]}{8\sqrt{14(35 - i\sqrt{7})}} \\ & - \frac{(355 + 73i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i + \sqrt{7} + 8ix}{\sqrt{2(35 + i\sqrt{7})}}\right]}{8\sqrt{14(35 + i\sqrt{7})}} - \frac{1}{16}(35 - 9i\sqrt{7}) \operatorname{Log}[x] - \frac{1}{16}(35 + 9i\sqrt{7}) \operatorname{Log}[x] + \\ & \frac{1}{32}(35 - 9i\sqrt{7}) \operatorname{Log}[4i + (i - \sqrt{7})x + 4ix^2] + \frac{1}{32}(35 + 9i\sqrt{7}) \operatorname{Log}[4i + (i + \sqrt{7})x + 4ix^2] \end{aligned}$$

Result (type 7, 116 leaves):

$$-\frac{5}{4x^2} + \frac{3}{4x} - \frac{35 \operatorname{Log}[x]}{8} + \frac{1}{8} \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4, \frac{61 \operatorname{Log}[x - \#1] + 141 \operatorname{Log}[x - \#1]\#1 + 47 \operatorname{Log}[x - \#1]\#1^2 + 70 \operatorname{Log}[x - \#1]\#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 257: Result is not expressed in closed-form.

$$\int \frac{x^2(3a + bx^2)}{a^2 + 2abx^2 + b^2x^4 + c^2x^6} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{cx^3}{a+bx^2}\right]}{c}$$

Result (type 7, 87 leaves):

$$\frac{1}{2} \operatorname{RootSum}\left[a^2 + 2ab\#1^2 + b^2\#1^4 + c^2\#1^6, \frac{3a \operatorname{Log}[x - \#1]\#1 + b \operatorname{Log}[x - \#1]\#1^3}{2ab + 2b^2\#1^2 + 3c^2\#1^4} \&\right]$$

Problem 387: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 - (1 + x^2)^4} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{i \sqrt{1 - i 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 - i 2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{i \sqrt{1 + i 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + i 2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{\sqrt{1 + 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + 2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{\sqrt{-1 + 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1 + 2^{1/4}}}\right]}{4 \times 2^{3/4}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8} \operatorname{RootSum}\left[-1 + 4 \#1^2 + 6 \#1^4 + 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{1 + 3 \#1^2 + 3 \#1^4 + \#1^6} \&\right]$$

Problem 388: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 - (1 - x^2)^4} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$-\frac{\sqrt{-1 + 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{-1 + 2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{i \sqrt{1 - i 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 - i 2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{i \sqrt{1 + i 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + i 2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{\sqrt{1 + 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + 2^{1/4}}}\right]}{4 \times 2^{3/4}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8} \operatorname{RootSum}\left[-1 - 4 \#1^2 + 6 \#1^4 - 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{-1 + 3 \#1^2 - 3 \#1^4 + \#1^6} \&\right]$$

Problem 389: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 + (1 + x^2)^4} dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\frac{(-1)^{1/4} \sqrt{1 - (-2)^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 - (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i (-2)^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + i (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} -$$

$$\frac{(-1)^{1/4} \sqrt{1 + (-2)^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{1}{8} i \left((-2)^{1/4} + \sqrt{2} \right) \sqrt{\frac{1 + i}{(1 + i) + 2^{3/4}}} \operatorname{ArcTan}\left[\sqrt{\frac{1 + i}{(1 + i) + 2^{3/4}}} x\right]$$

Result (type 7, 61 leaves):

$$\frac{1}{8} \operatorname{RootSum}\left[3 + 4 \#1^2 + 6 \#1^4 + 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{1 + 3 \#1^2 + 3 \#1^4 + \#1^6} \&\right]$$

Problem 390: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 + (1 - x^2)^4} dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$- \frac{(-1)^{1/4} \sqrt{1 - (-2)^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 - (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i (-2)^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + i (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} +$$

$$\frac{(-1)^{1/4} \sqrt{1 + (-2)^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{1}{8} i \left((-2)^{1/4} + \sqrt{2} \right) \sqrt{\frac{1 + i}{(1 + i) + 2^{3/4}}} \operatorname{ArcTanh}\left[\sqrt{\frac{1 + i}{(1 + i) + 2^{3/4}}} x\right]$$

Result (type 7, 61 leaves):

$$\frac{1}{8} \operatorname{RootSum}\left[3 - 4 \#1^2 + 6 \#1^4 - 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{-1 + 3 \#1^2 - 3 \#1^4 + \#1^6} \&\right]$$

Problem 391: Result is not expressed in closed-form.

$$\int \frac{1 - x^2}{a + b (1 - x^2)^4} dx$$

Optimal (type 3, 663 leaves, 16 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{(-a)^{1/4} - b^{1/4}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} - b^{1/4}} b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} - \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} + \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \frac{\text{ArcTanh}\left[\frac{b^{1/8} x}{(-a)^{1/4} + b^{1/4}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} + b^{1/4}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{2}} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} - \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt{2}} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}}
\end{aligned}$$

Result (type 7, 63 leaves):

$$\frac{\text{RootSum}\left[a + b - 4 b \#1^2 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \ \&, \frac{\text{Log}[x - \#1]}{\#1 - 2 \#1^3 + \#1^5} \ \&\right]}{8 b}$$

Problem 392: Result is not expressed in closed-form.

$$\int \frac{1 - x^2}{a + b (-1 + x^2)^4} dx$$

Optimal (type 3, 663 leaves, 17 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} - b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} - b^{1/4}} b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} - \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} + \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} + b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} + b^{1/4}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{2}} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} - \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt{2}} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}}
\end{aligned}$$

Result (type 7, 63 leaves):

$$\frac{\text{RootSum}\left[a + b - 4 b \#1^2 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{\text{Log}[x - \#1]}{\#1 - 2 \#1^3 + \#1^5} \&\right]}{8 b}$$

Problem 393: Result is not expressed in closed-form.

$$\int \frac{(1 + x^2)^2}{a x^6 + b (1 + x^2)^3} dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} b^{5/6}}$$

Result (type 7, 95 leaves):

$$\frac{1}{6} \text{RootSum}\left[b + 3 b \#1^2 + 3 b \#1^4 + a \#1^6 + b \#1^6 \&, \frac{\text{Log}[x - \#1] + 2 \text{Log}[x - \#1] \#1^2 + \text{Log}[x - \#1] \#1^4}{b \#1 + 2 b \#1^3 + a \#1^5 + b \#1^5} \&\right]$$

Problem 421: Result more than twice size of optimal antiderivative.

$$\int \frac{2}{-1 + 4x^2} dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}[2x]$$

Result (type 3, 23 leaves):

$$2 \left(\frac{1}{4} \text{Log}[1 - 2x] - \frac{1}{4} \text{Log}[1 + 2x] \right)$$

Problem 491: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{1 + (-1 + x^2)^2} dx$$

Optimal (type 3, 188 leaves, 10 steps):

$$-\frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} - 2x}{\sqrt{2(-1 + \sqrt{2})}}\right] +$$

$$\frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} + 2x}{\sqrt{2(-1 + \sqrt{2})}}\right] + \frac{\text{Log}[\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2]}{4\sqrt{2(1 + \sqrt{2})}} - \frac{\text{Log}[\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2]}{4\sqrt{2(1 + \sqrt{2})}}$$

Result (type 3, 39 leaves):

$$-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1-i}}\right]}{(-1-i)^{3/2}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1+i}}\right]}{(-1+i)^{3/2}}$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}}\right] + \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3\sqrt{3} + \frac{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}{3\sqrt{3}}}}$$

Result (type 4, 148 leaves):

$$\frac{4i\sqrt{2} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(1+2 \times 2^{2/3} - i\sqrt{3}) \sqrt{1+x^3}}$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{1-x^3}}\right] - \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3\sqrt{3} - \frac{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}{3\sqrt{3}}}}$$

Result (type 4, 148 leaves):

$$\frac{4i\sqrt{2} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(1+2 \times 2^{2/3} - i\sqrt{3}) \sqrt{1-x^3}}$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{-1+x^3}}\right]}{3\sqrt{3}} - \frac{2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 146 leaves):

$$\frac{4i\sqrt{2} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(1+2 \times 2^{2/3} - i\sqrt{3}) \sqrt{-1+x^3}}$$

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{-1-x^3}}\right]}{3\sqrt{3}} + \frac{2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 150 leaves):

$$\frac{4i\sqrt{2} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(1+2 \times 2^{2/3} - i\sqrt{3}) \sqrt{-1-x^3}}$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 280 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a + b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \frac{2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3}}$$

Result (type 4, 164 leaves):

$$\frac{2 i \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{a + b x^3}}$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} - \frac{2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3}}$$

Result (type 4, 166 leaves):

$$\frac{2 i \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{a - b x^3}}$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \frac{2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3}}$$

Result (type 4, 167 leaves):

$$\frac{2 i \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{-a + b x^3}}$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 293 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \frac{2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3}}$$

Result (type 4, 167 leaves):

$$\frac{2 i \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{-a - b x^3}}$$

Problem 9: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c + dx) \sqrt{c^3 + 4d^3 x^3}} dx$$

Optimal (type 4, 249 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3}-\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right]}{3\sqrt{3}c^{3/2}d} + \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (c+2^{2/3}dx) \sqrt{\frac{c^2-2^{2/3}cdx+2 \cdot 2^{1/3}d^2x^2}{((1+\sqrt{3})c+2^{2/3}dx)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})c+2^{2/3}dx}{(1+\sqrt{3})c+2^{2/3}dx}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} cd \sqrt{\frac{c(c+2^{2/3}dx)}{((1+\sqrt{3})c+2^{2/3}dx)^2}} \sqrt{c^3+4d^3x^3}}$$

Result (type 4, 169 leaves):

$$-\left(\left(\left(i 2^{5/6} \sqrt{\frac{2^{1/3}c+2dx}{(1+(-1)^{1/3})c}} \sqrt{2^{2/3}-\frac{2 \times 2^{1/3}dx}{c} + \frac{4d^2x^2}{c^2}} \operatorname{EllipticPi}\left[\frac{i 2^{1/3}\sqrt{3}}{2+(-2)^{1/3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3}c+2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}}}{2^{1/6}}\right], (-1)^{1/3}\right]\right) / \left(\left(2+(-2)^{1/3}\right) d \sqrt{c^3+4d^3x^3}\right)\right)$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 146 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3(3+2\sqrt{3})}} + \frac{\sqrt{2+\sqrt{3}}(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 136 leaves):

$$\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(3i+(1+2i)\sqrt{3})\sqrt{1+x^3}}$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right]-\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})}3^{3/4}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}}$$

Result (type 4, 136 leaves):

$$\frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(3i+(1+2i)\sqrt{3})\sqrt{1-x^3}}$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 167 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right]-\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})}3^{3/4}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

Result (type 4, 134 leaves):

$$\frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(3i+(1+2i)\sqrt{3})\sqrt{-1+x^3}}$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 157 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3(3+2\sqrt{3})}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{3^{3/4}\sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}}\sqrt{-1-x^3}}$$

Result (type 4, 138 leaves):

$$\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(3i+(1+2i)\sqrt{3})\sqrt{-1-x^3}}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\frac{(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right] + 2\sqrt{26+15\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} + 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} +$$

$$\frac{4 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56\sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 128 leaves):

$$\frac{4\sqrt{2} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{7i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(7i+\sqrt{3}) \sqrt{1+x^3}}$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 382 leaves, 8 steps):

$$\frac{(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right] + 2\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{2\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} + 3^{1/4} (4+\sqrt{3}) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} +$$

$$\frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} (553+304\sqrt{3}), -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 128 leaves):

$$\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{5i+\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]}{(5i+\sqrt{3})\sqrt{1-x^3}}$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 376 leaves, 8 steps):

$$\frac{(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\operatorname{ArcTanh}\left[\frac{\sqrt{7}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right] + 2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{2\sqrt{7}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3} + 13\times 3^{1/4}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} +$$

$$\frac{4\times 3^{1/4}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\operatorname{EllipticPi}\left[\frac{1}{169}(553+304\sqrt{3}),-\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{13\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

Result (type 4, 126 leaves):

$$\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{5i+\sqrt{3}},\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right],\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]}{(5i+\sqrt{3})\sqrt{-1+x^3}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 342 leaves, 8 steps):

$$\frac{(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right] + 2(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \frac{3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

$$\frac{4 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56\sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 130 leaves):

$$\frac{4\sqrt{2} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{7i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]}{(7i+\sqrt{3}) \sqrt{-1-x^3}}$$

Problem 18: Unable to integrate problem.

$$\int \frac{1}{(c+dx) (-c^3+d^3x^3)^{1/3}} dx$$

Optimal (type 3, 139 leaves, 1 step):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3}(c-dx)}{(-c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} c d} + \frac{\operatorname{Log}\left[(c-dx)(c+dx)^2\right]}{4 \times 2^{1/3} c d} - \frac{3 \operatorname{Log}\left[d(c-dx) + 2^{2/3} d (-c^3+d^3x^3)^{1/3}\right]}{4 \times 2^{1/3} c d}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(c+dx) (-c^3+d^3x^3)^{1/3}} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{1}{(c+dx) (2c^3+d^3x^3)^{1/3}} dx$$

Optimal (type 3, 186 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2dx}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}cd} - \frac{\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2(2c+dx)}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2cd} - \frac{\text{Log}[c+dx]}{2cd} - \frac{\text{Log}[-dx+(2c^3+d^3x^3)^{1/3}]}{4cd} + \frac{3\text{Log}[d(2c+dx)-d(2c^3+d^3x^3)^{1/3}]}{4cd}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{1/3}} dx$$

Problem 20: Unable to integrate problem.

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Optimal (type 3, 187 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2dx}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2(2c+dx)}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2c^2d} - \frac{\text{Log}[c+dx]}{2c^2d} - \frac{\text{Log}[dx-(2c^3+d^3x^3)^{1/3}]}{4c^2d} + \frac{3\text{Log}[d(2c+dx)-d(2c^3+d^3x^3)^{1/3}]}{4c^2d}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Problem 21: Unable to integrate problem.

$$\int \frac{1}{(1+2^{1/3}x)(1+x^3)^{2/3}} dx$$

Optimal (type 3, 147 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2x}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2(2^{2/3}+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} - \frac{\text{Log}[1+2^{1/3}x]}{2^{2/3}} - \frac{\text{Log}[x-(1+x^3)^{1/3}]}{2 \times 2^{2/3}} + \frac{3\text{Log}[2+2^{1/3}x-2^{1/3}(1+x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 8, 23 leaves):

$$\int \frac{1}{(1+2^{1/3}x)(1+x^3)^{2/3}} dx$$

Problem 22: Unable to integrate problem.

$$\int \frac{1}{(1 - 2^{1/3} x) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 159 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2^{2/3} - 2x}{\sqrt{3}(1-x^3)^{1/3}}\right]}{2^{2/3}} + \frac{\operatorname{ArcTan}\left[\frac{1 - 2x}{\sqrt{3}(1-x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{Log}[1 - 2^{1/3} x]}{2^{2/3}} + \frac{\operatorname{Log}[-x - (1 - x^3)^{1/3}]}{2 \times 2^{2/3}} - \frac{3 \operatorname{Log}[-2 + 2^{1/3} x + 2^{1/3} (1 - x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 8, 26 leaves):

$$\int \frac{1}{(1 - 2^{1/3} x) (1 - x^3)^{2/3}} dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{(a + b x^3)^{1/3}}{c + d x} dx$$

Optimal (type 6, 435 leaves, 13 steps):

$$\begin{aligned} & \frac{(a + b x^3)^{1/3}}{d} + \frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c \left(1 + \frac{b x^3}{a}\right)^{1/3}} + \frac{b^{1/3} c \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} - \\ & \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}[c^3 + d^3 x^3]}{3 d^2} + \\ & \frac{b^{1/3} c \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 d^2} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b x^3)^{1/3}}{c + d x} dx$$

Problem 28: Unable to integrate problem.

$$\int \frac{(a + b x^3)^{1/3}}{(c + d x)^2} dx$$

Optimal (type 6, 818 leaves, 20 steps):

$$\begin{aligned} & -\frac{c^2 (a + b x^3)^{1/3}}{d (c^3 + d^3 x^3)} - \frac{d x^2 (a + b x^3)^{1/3}}{c^3 + d^3 x^3} + \frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 2, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^2 \left(1 + \frac{b x^3}{a}\right)^{1/3}} - \frac{d^3 x^4 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^5 \left(1 + \frac{b x^3}{a}\right)^{1/3}} \\ & - \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{2 a d \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b c^2 \operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c^3 - a d^3)^{2/3}} \\ & - \frac{b c^2 \operatorname{Log}[c^3 + d^3 x^3]}{6 d^2 (b c^3 - a d^3)^{2/3}} - \frac{a d \operatorname{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{2/3}} - \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}[c^3 + d^3 x^3]}{18 c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b^{1/3} \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d^2} + \\ & - \frac{a d \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c d^2 (b c^3 - a d^3)^{2/3}} + \frac{b c^2 \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 d^2 (b c^3 - a d^3)^{2/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b x^3)^{1/3}}{(c + d x)^2} dx$$

Problem 33: Unable to integrate problem.

$$\int \frac{1}{(c + d x) (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 333 leaves, 10 steps):

$$\begin{aligned} & -\frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^2 (a + b x^3)^{1/3}} + \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} \\ & - \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} + \frac{\operatorname{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{1/3}} - \frac{\operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} - \frac{\operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x) (a + b x^3)^{1/3}} dx$$

Problem 34: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 761 leaves, 17 steps):

$$\begin{aligned} & \frac{c^2 d^2 (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{c d^3 x (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 2, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a + b x^3)^{1/3}} + \\ & \frac{d^4 x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^6 (a + b x^3)^{1/3}} + \frac{2 a d^3 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} - \\ & \frac{b c^2 \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{4/3}} + \frac{b c^2 \text{Log}[c^3 + d^3 x^3]}{6 (b c^3 - a d^3)^{4/3}} + \frac{a d^3 \text{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 c (b c^3 - a d^3)^{4/3}} - \\ & \frac{a d^3 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{4/3}} - \frac{(3 b c^3 - 2 a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c (b c^3 - a d^3)^{4/3}} - \frac{b c^2 \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{4/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{1/3}} dx$$

Problem 35: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 1513 leaves, 32 steps):

$$\begin{aligned}
& \frac{3 c^4 d^2 (a+b x^3)^{2/3}}{2 (b c^3-a d^3) (c^3+d^3 x^3)^2} - \frac{3 c^3 d^3 x (a+b x^3)^{2/3}}{2 (b c^3-a d^3) (c^3+d^3 x^3)^2} + \frac{4 b c^4 d^2 (a+b x^3)^{2/3}}{3 (b c^3-a d^3)^2 (c^3+d^3 x^3)} - \frac{c d^2 (b c^3-3 a d^3) (a+b x^3)^{2/3}}{3 (b c^3-a d^3)^2 (c^3+d^3 x^3)} + \\
& \frac{d^3 (3 b c^3-7 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)} - \frac{d^3 (9 b c^3-5 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)} - \frac{7 d^3 (3 b c^3+a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)} - \\
& \frac{3 d x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 3, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^4 (a+b x^3)^{1/3}} + \frac{6 d^4 x^5 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 3, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^7 (a+b x^3)^{1/3}} + \frac{2 a^2 d^6 \operatorname{ArcTan}\left[\frac{1+\frac{2(b c^3-a d^3)^{1/3} x}{c(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3-a d^3) \operatorname{ArcTan}\left[\frac{1+\frac{2(b c^3-a d^3)^{1/3} x}{c(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}} + \frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \operatorname{ArcTan}\left[\frac{1+\frac{2(b c^3-a d^3)^{1/3} x}{c(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}} - \frac{4 b^2 c^4 \operatorname{ArcTan}\left[\frac{1-\frac{2 d(a+b x^3)^{1/3}}{(b c^3-a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3-a d^3)^{7/3}} + \\
& \frac{b c (b c^3-3 a d^3) \operatorname{ArcTan}\left[\frac{1-\frac{2 d(a+b x^3)^{1/3}}{(b c^3-a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3-a d^3)^{7/3}} + \frac{2 b^2 c^4 \operatorname{Log}\left[c^3+d^3 x^3\right]}{9 (b c^3-a d^3)^{7/3}} + \frac{a^2 d^6 \operatorname{Log}\left[c^3+d^3 x^3\right]}{27 c^2 (b c^3-a d^3)^{7/3}} - \frac{b c (b c^3-3 a d^3) \operatorname{Log}\left[c^3+d^3 x^3\right]}{18 (b c^3-a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3-a d^3) \operatorname{Log}\left[c^3+d^3 x^3\right]}{54 c^2 (b c^3-a d^3)^{7/3}} + \frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \operatorname{Log}\left[c^3+d^3 x^3\right]}{54 c^2 (b c^3-a d^3)^{7/3}} - \frac{a^2 d^6 \operatorname{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{9 c^2 (b c^3-a d^3)^{7/3}} - \\
& \frac{7 a d^3 (3 b c^3-a d^3) \operatorname{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{18 c^2 (b c^3-a d^3)^{7/3}} - \frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \operatorname{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{18 c^2 (b c^3-a d^3)^{7/3}} - \\
& \frac{2 b^2 c^4 \operatorname{Log}\left[(b c^3-a d^3)^{1/3}+d(a+b x^3)^{1/3}\right]}{3 (b c^3-a d^3)^{7/3}} + \frac{b c (b c^3-3 a d^3) \operatorname{Log}\left[(b c^3-a d^3)^{1/3}+d(a+b x^3)^{1/3}\right]}{6 (b c^3-a d^3)^{7/3}}
\end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c+d x)^3 (a+b x^3)^{1/3}} dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(c+d x) (a+b x^3)^{2/3}} dx$$

Optimal (type 6, 332 leaves, 10 steps):

$$\frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right]}{c (a + bx^3)^{2/3}} + \frac{d \text{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} -$$

$$\frac{d \text{ArcTan}\left[\frac{1 - \frac{2d(a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} - \frac{d \text{Log}\left[c^3 + d^3 x^3\right]}{3 (b c^3 - a d^3)^{2/3}} + \frac{d \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}} + \frac{d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + dx) (a + bx^3)^{2/3}} dx$$

Problem 41: Unable to integrate problem.

$$\int \frac{1}{(c + dx)^2 (a + bx^3)^{2/3}} dx$$

Optimal (type 6, 760 leaves, 18 steps):

$$\frac{c^2 d^2 (a + bx^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{d^4 x^2 (a + bx^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 2, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right]}{c^2 (a + bx^3)^{2/3}} -$$

$$\frac{d^3 x^4 \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right]}{2 c^5 (a + bx^3)^{2/3}} + \frac{2 a d^4 \text{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} + \frac{2 d (3 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} -$$

$$\frac{2 b c^2 d \text{ArcTan}\left[\frac{1 - \frac{2d(a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{5/3}} - \frac{b c^2 d \text{Log}\left[c^3 + d^3 x^3\right]}{3 (b c^3 - a d^3)^{5/3}} - \frac{a d^4 \text{Log}\left[c^3 + d^3 x^3\right]}{9 c (b c^3 - a d^3)^{5/3}} - \frac{d (3 b c^3 - a d^3) \text{Log}\left[c^3 + d^3 x^3\right]}{9 c (b c^3 - a d^3)^{5/3}} +$$

$$\frac{a d^4 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{d (3 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{b c^2 d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{(b c^3 - a d^3)^{5/3}}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + dx)^2 (a + bx^3)^{2/3}} dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}} dx$$

Optimal (type 6, 1357 leaves, 30 steps):

$$\begin{aligned} & \frac{3c^4 d^2 (a+bx^3)^{1/3}}{2(b c^3 - a d^3)(c^3 + d^3 x^3)^2} + \frac{3c^2 d^4 x^2 (a+bx^3)^{1/3}}{2(b c^3 - a d^3)(c^3 + d^3 x^3)^2} + \frac{5b c^4 d^2 (a+bx^3)^{1/3}}{3(b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{c d^2 (b c^3 - 6 a d^3) (a+bx^3)^{1/3}}{6(b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \\ & \frac{d^4 (9b c^3 - 4 a d^3) x^2 (a+bx^3)^{1/3}}{6c(b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{d^4 (3b c^3 + 2 a d^3) x^2 (a+bx^3)^{1/3}}{3c(b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 3, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a+bx^3)^{2/3}} - \\ & \frac{7 d^3 x^4 \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 3, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{4 c^6 (a+bx^3)^{2/3}} + \frac{d^6 x^7 \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 3, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{7 c^9 (a+bx^3)^{2/3}} + \\ & \frac{2 a d^4 (6 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} - \\ & \frac{10 b^2 c^4 d \text{ArcTan}\left[\frac{1 - \frac{2 d (a+bx^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} + \frac{b c d (b c^3 - 6 a d^3) \text{ArcTan}\left[\frac{1 - \frac{2 d (a+bx^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} - \frac{5 b^2 c^4 d \text{Log}[c^3 + d^3 x^3]}{9 (b c^3 - a d^3)^{8/3}} + \\ & \frac{b c d (b c^3 - 6 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 (b c^3 - a d^3)^{8/3}} - \frac{a d^4 (6 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{9 c^2 (b c^3 - a d^3)^{8/3}} - \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}[c^3 + d^3 x^3]}{18 c^2 (b c^3 - a d^3)^{8/3}} + \\ & \frac{a d^4 (6 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+bx^3)^{1/3}\right]}{3 c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+bx^3)^{1/3}\right]}{6 c^2 (b c^3 - a d^3)^{8/3}} + \\ & \frac{5 b^2 c^4 d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+bx^3)^{1/3}\right]}{3 (b c^3 - a d^3)^{8/3}} - \frac{b c d (b c^3 - 6 a d^3) \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+bx^3)^{1/3}\right]}{6 (b c^3 - a d^3)^{8/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}} dx$$

Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 326 leaves):

$$\begin{aligned} & - \left(\left(4 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \right. \\ & \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left(6i+3i2^{1/3}-2\sqrt{3}+2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \right. \\ & \left. \left. 6i\sqrt{3} \sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) \right) / \\ & \left(\sqrt{3} \left(1+2 \times 2^{2/3} - i\sqrt{3} \right) \sqrt{i+\sqrt{3}-2ix} \sqrt{1+x^3} \right) \end{aligned}$$

Problem 44: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x) \sqrt{1-x^3}} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{1-x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 327 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \right. \\
& \quad \left(\sqrt{-i+\sqrt{3}-2ix} \left(-6i-3i2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i+\sqrt{3}} \right] + \right. \\
& \quad \left. \left. 6i\sqrt{3} \sqrt{i+\sqrt{3}+2ix} \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i+\sqrt{3}} \right] \right) \right) / \\
& \quad \left(\sqrt{3} \left(1+2 \times 2^{2/3} - i\sqrt{3} \right) \sqrt{i+\sqrt{3}+2ix} \sqrt{1-x^3} \right) \Big)
\end{aligned}$$

Problem 45: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$-\frac{2 \times 2^{2/3} \text{ArcTanh} \left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{-1+x^3}} \right]}{\sqrt{3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \right. \\
& \quad \left(\sqrt{-i+\sqrt{3}-2ix} \left(-6i-3i2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i+\sqrt{3}} \right] + \right. \\
& \quad \left. \left. 6i\sqrt{3} \sqrt{i+\sqrt{3}+2ix} \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i+\sqrt{3}} \right] \right) \right) / \\
& \quad \left(\sqrt{3} \left(1+2 \times 2^{2/3} - i\sqrt{3} \right) \sqrt{i+\sqrt{3}+2ix} \sqrt{-1+x^3} \right) \Big)
\end{aligned}$$

Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 39 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 328 leaves):

$$\begin{aligned} & - \left(\left(4 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \right. \\ & \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left(6i+3i2^{1/3}-2\sqrt{3}+2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \right. \\ & \left. \left. 6i\sqrt{3} \sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \\ & \left. \left(\sqrt{3} (1+2 \times 2^{2/3} - i\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{-1-x^3} \right) \right) \end{aligned}$$

Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3}+2^{1/3} b^{1/3} x)}{\sqrt{a+bx^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 325 leaves):

$$\frac{1}{\sqrt{3} b^{1/3} \sqrt{a + b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{2 \times 3^{1/4} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} - \frac{1}{(-1)^{1/3} + 2^{2/3}} \right.$$

$$\left. 3 (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 48: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 336 leaves):

$$\frac{1}{b^{1/3} \sqrt{a - b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{2 \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right. \\ \left. (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{3 + \frac{3 b^{1/3} x}{a^{1/3}} + \frac{3 b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 390 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left(2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} + (-1)^{1/3} b^{1/3} x \right)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] - \right. \right. \\
& \left. \left. (-1)^{1/3} 2^{2/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \left. \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right) \right)
\end{aligned}$$

Problem 50: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left(-\frac{1}{3^{1/4}} 2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right. \right. \\
& \left. \left. (-1)^{1/3} 2^{2/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right) \right)
\end{aligned}$$

Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 d x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{3} \sqrt{c} (c + 2 d x)}{\sqrt{c^3 + 4 d^3 x^3}} \right]}{\sqrt{3} \sqrt{c} d}$$

Result (type 4, 373 leaves):

$$\left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
\left. 2 \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 \left((-1)^{1/3} + 2^{2/3} \right) d x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] - \right. \\
\left. (-1)^{1/3} 2^{2/3} \sqrt{3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
\left. \text{EllipticPi} \left[\frac{i 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] \right) / \left((2 + (-2)^{1/3}) d \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)$$

Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{2 + 3x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 158 leaves, 4 steps):

$$\frac{2 (2 - 3 \times 2^{2/3}) \text{ArcTan} \left[\frac{\sqrt{3} (1 + 2^{1/3} x)}{\sqrt{1 + x^3}} \right]}{3 \sqrt{3}} + \frac{2 (3 + 2 \times 2^{1/3}) \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4 \sqrt{3} \right]}{3 \times 3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}$$

Result (type 4, 336 leaves):

$$\left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(3 \sqrt{-i+\sqrt{3}+2ix} \left(-6-3 \times 2^{1/3}-2i\sqrt{3}+i2^{1/3}\sqrt{3}+(3 \times 2^{1/3}+4i\sqrt{3}+i2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \right. \\ \left. \left. 4\sqrt{3}(-3+2^{1/3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right) \right) / \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3} \right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$-\frac{2(2+3 \times 2^{2/3}) \text{ArcTan}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{1-x^3}}\right]}{3\sqrt{3}} + \frac{2(3-2 \times 2^{1/3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 335 leaves):

$$\left(2 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \\ \left(-3i\sqrt{-i+\sqrt{3}-2ix} \left(-6i-3i2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3}+(-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \\ \left. 4\sqrt{3}(3+2^{1/3})\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right) \right) / \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3} \right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3x}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 176 leaves, 4 steps):

$$\frac{2(2 + 3 \times 2^{2/3}) \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1 - 2^{1/3}x)}{\sqrt{-1 + x^3}}\right]}{3\sqrt{3}} + \frac{2(3 - 2 \times 2^{1/3}) \sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x}\right], -7 + 4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3}}$$

Result (type 4, 333 leaves):

$$\left(2 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(-3i \sqrt{-i+\sqrt{3}-2ix} \left(-6i-3i2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 4\sqrt{3}(3+2^{1/3}) \sqrt{i+\sqrt{3}+2ix} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right) \right) / \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3}) \sqrt{i+\sqrt{3}+2ix} \sqrt{-1+x^3} \right)$$

Problem 55: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 169 leaves, 4 steps):

$$\frac{2(2 - 3 \times 2^{2/3}) \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1 + 2^{1/3}x)}{\sqrt{-1 - x^3}}\right]}{3\sqrt{3}} + \frac{2(3 + 2 \times 2^{1/3}) \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}$$

Result (type 4, 338 leaves):

$$\left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(3 \sqrt{-i+\sqrt{3}+2ix} \left(-6-3 \times 2^{1/3} - 2i\sqrt{3} + i2^{1/3}\sqrt{3} + (3 \times 2^{1/3} + 4i\sqrt{3} + i2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \right. \\ \left. \left. 4\sqrt{3}(-3+2^{1/3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right] \right) \Bigg/ \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3} \right)$$

Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 159 leaves, 4 steps):

$$\frac{2(e-2^{2/3}f)\text{ArcTan}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}}\right] + \frac{2\sqrt{2+\sqrt{3}}(2^{1/3}e+f)(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3\sqrt{3} + \frac{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}{3\sqrt{3}}}}$$

Result (type 4, 340 leaves):

$$\left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left(f \sqrt{-i+\sqrt{3}+2ix} \left(-6-3 \times 2^{1/3} - 2i\sqrt{3} + i2^{1/3}\sqrt{3} + (3 \times 2^{1/3} + 4i\sqrt{3} + i2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \\ \left. 2\sqrt{3}(2^{1/3}e-2f)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right] \right) \Bigg/ \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3} \right)$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 175 leaves, 4 steps):

$$\frac{2 (e + 2^{2/3} f) \operatorname{ArcTan}\left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{1 - x^3}}\right] - 2 \sqrt{2 + \sqrt{3}} (2^{1/3} e - f) (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x}\right], -7 - 4 \sqrt{3}\right]}{3 \sqrt{3} \sqrt{3} \times 3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3}}$$

Result (type 4, 340 leaves):

$$\left(2 \times 2^{1/6} \sqrt{-\frac{i (-1 + x)}{3 i + \sqrt{3}}} \left(-i f \sqrt{-i + \sqrt{3} - 2 i x} \left(-6 i - 3 i 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + (-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3}) x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} + 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] + 2 \sqrt{3} (2^{1/3} e + 2 f) \sqrt{i + \sqrt{3} + 2 i x} \sqrt{1 + x + x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} + 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \right) / \left(\sqrt{3} (i + 2 i 2^{2/3} + \sqrt{3}) \sqrt{i + \sqrt{3} + 2 i x} \sqrt{1 - x^3} \right)$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 178 leaves, 4 steps):

$$\frac{2 (e + 2^{2/3} f) \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{-1 + x^3}}\right] - 2 \sqrt{2 - \sqrt{3}} (2^{1/3} e - f) (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x}\right], -7 + 4 \sqrt{3}\right]}{3 \sqrt{3} \sqrt{3} \times 3^{1/4} \sqrt{-\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3}}$$

Result (type 4, 338 leaves):

$$\left(2 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(-i f \sqrt{-i+\sqrt{3}-2ix} \left(-6i-3i2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3} + (-3i2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 2\sqrt{3}(2^{1/3}e+2f)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right] \right) / \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}+2ix}\sqrt{-1+x^3} \right)$$

Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\frac{2(e-2^{2/3}f)\text{ArcTanh}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{-1-x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(2^{1/3}e+f)(1+x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 342 leaves):

$$\left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left(f \sqrt{-i+\sqrt{3}+2ix} \left(-6-3 \times 2^{1/3}-2i\sqrt{3}+i2^{1/3}\sqrt{3} + (3 \times 2^{1/3}+4i\sqrt{3}+i2^{1/3}\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] - \right. \\ \left. \left. 2\sqrt{3}(2^{1/3}e-2f)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right] \right) / \\ \left(\sqrt{3}(i+2i2^{2/3}+\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3} \right)$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 316 leaves, 4 steps):

$$\frac{2 (b^{1/3} e - 2^{2/3} a^{1/3} f) \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a + b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} +$$

$$\left(2 \sqrt{2 + \sqrt{3}} (2^{1/3} b^{1/3} e + a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 336 leaves):

$$\frac{1}{\sqrt{3} b^{2/3} \sqrt{a + b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{3^{1/4} f \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right.$$

$$\left. (-1)^{1/3} (1 + (-1)^{1/3}) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 324 leaves, 4 steps):

$$\frac{2 (b^{1/3} e + 2^{2/3} a^{1/3} f) \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{2/3}}$$

$$\left(2 \sqrt{2 + \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)$$

Result (type 4, 399 leaves):

$$\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right.$$

$$\left. - \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 333 leaves, 4 steps):

$$\frac{2 (b^{1/3} e + 2^{2/3} a^{1/3} f) \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}}$$

$$\left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)$$

Result (type 4, 400 leaves):

$$\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right.$$

$$\left. - \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right.$$

$$\left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 329 leaves, 4 steps):

$$\frac{2 (b^{1/3} e - 2^{2/3} a^{1/3} f) \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} +$$

$$\left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e + a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4 \sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)$$

Result (type 4, 387 leaves):

$$\left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{3^{1/4}} ((-1)^{1/3} + 2^{2/3}) f ((-1)^{1/3} a^{1/3} - b^{1/3} x) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(((-1)^{1/3} + 2^{2/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 265 leaves, 4 steps):

$$\frac{2 (d e - c f) \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c+2 d x)}{\sqrt{c^3+4 d^3 x^3}}\right]}{3 \sqrt{3} c^{3/2} d^2} +$$

$$\left(2^{1/3} \sqrt{2+\sqrt{3}} (2 d e + c f) (c+2^{2/3} d x) \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \times 2^{1/3} d^2 x^2}{\left((1+\sqrt{3}) c + 2^{2/3} d x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) c + 2^{2/3} d x}{(1+\sqrt{3}) c + 2^{2/3} d x}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c+2^{2/3} d x)}{\left((1+\sqrt{3}) c + 2^{2/3} d x\right)^2}} \sqrt{c^3+4 d^3 x^3} \right)$$

Result (type 4, 380 leaves):

$$\left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1+(-1)^{1/3}) c}} \right.$$

$$\left. - f \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1+(-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 \left((-1)^{1/3} + 2^{2/3} \right) d x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1+(-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1+(-1)^{1/3}) (-d e + c f) \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1+(-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1+(-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}\right] \right) / \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1+(-1)^{1/3}) c}} \sqrt{c^3+4 d^3 x^3} \right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 207 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{i 2^{2/3} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3}+2^{2/3}} \right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{1-x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 209 leaves):

$$\frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i 2^{2/3} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3} + 2^{2/3}} \right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} - x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{-1+x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 207 leaves):

$$\frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i 2^{2/3} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3} + 2^{2/3}} \right)$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} + x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{-1-x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 209 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{i 2^{2/3} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3}+2^{2/3}} \right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}a^{1/3}+b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}a^{1/6}(a^{1/3}+2^{1/3}b^{1/3}x)}{\sqrt{a+bx^3}}\right]}{3\sqrt{3}a^{1/6}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}(a^{1/3}+b^{1/3}x)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3}}$$

Result (type 4, 324 leaves):

$$\frac{1}{\sqrt{3} b^{2/3} \sqrt{a + b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{3^{1/4} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right.$$

$$\left. (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$- \frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \frac{2 \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3}}$$

Result (type 4, 388 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] - \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 292 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}} \sqrt{-a + b x^3}}$$

Result (type 4, 389 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] - \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}} \sqrt{-a - b x^3}}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \frac{1}{3^{1/4}} \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] + \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} \right. \\
& \left. \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) \Bigg) / \\
& \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 246 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} - \sqrt{c} (c + 2 d x)}{\sqrt{c^3 + 4 d^3 x^3}}\right]}{3 \sqrt{3} \sqrt{c} d^2} + \frac{2^{1/3} \sqrt{2 + \sqrt{3}} (c + 2^{2/3} d x) \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \cdot 2^{1/3} d^2 x^2}{((1 + \sqrt{3}) c + 2^{2/3} d x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c + 2^{2/3} d x}{(1 + \sqrt{3}) c + 2^{2/3} d x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} d^2 \sqrt{\frac{c (c + 2^{2/3} d x)}{((1 + \sqrt{3}) c + 2^{2/3} d x)^2}} \sqrt{c^3 + 4 d^3 x^3}}
\end{aligned}$$

Result (type 4, 372 leaves):

$$\left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
\left. - \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 \left((-1)^{1/3} + 2^{2/3} \right) d x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] + \right. \\
\left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
\left. \left. \text{EllipticPi} \left[\frac{i 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] \right) \right) / \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)$$

Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{2}{3} \text{ArcTanh} \left[\frac{(1+x)^2}{3\sqrt{1+x^3}} \right]$$

Result (type 4, 265 leaves):

$$\left(2\sqrt{6} \sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}} \left(-i\sqrt{i+\sqrt{3}-2ix} (-i-\sqrt{3}+(-i+\sqrt{3})x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3}\sqrt{-i+\sqrt{3}+2ix} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \left((3i+\sqrt{3})\sqrt{-i+\sqrt{3}+2ix}\sqrt{1+x^3} \right)$$

Problem 75: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$-\frac{2}{3} \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right]$$

Result (type 4, 262 leaves):

$$-\left(\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2ix} (-1+i\sqrt{3}+x+i\sqrt{3}x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3}\sqrt{-i+\sqrt{3}-2ix} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \left((3i+\sqrt{3})\sqrt{-i+\sqrt{3}-2ix}\sqrt{1-x^3} \right) \right)$$

Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$-\frac{2}{3} \operatorname{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right]$$

Result (type 4, 260 leaves):

$$- \left(\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2ix} (-1+i\sqrt{3}+x+i\sqrt{3}x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3} \sqrt{-i+\sqrt{3}-2ix} \right. \right. \right. \\ \left. \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right) \right) \right) / \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{-1+x^3} \right)$$

Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{2}{3} \operatorname{ArcTan}\left[\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right]$$

Result (type 4, 267 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{-3i+\sqrt{3}}} \left(-i\sqrt{i+\sqrt{3}-2ix} (-i-\sqrt{3}+(-i+\sqrt{3})x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3} \sqrt{-i+\sqrt{3}+2ix} \right. \right. \\ \left. \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right) \right) \right) / \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}+2ix} \sqrt{-1-x^3} \right)$$

Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3}x}{(2a^{1/3} - b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{(a^{1/3}+b^{1/3}x)^2}{3a^{1/6}\sqrt{a+bx^3}}\right]}{3a^{1/6}b^{1/3}}$$

Result (type 4, 407 leaves):

$$\left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
\left. - \frac{1}{2\sqrt{2}} 3^{1/4} \left((i + \sqrt{3}) a^{1/3} - (-i + \sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + 3i \right. \\
\left. a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right) / \\
\left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)$$

Problem 79: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 52 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 370 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \quad \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}, (-1)^{1/3} \right] + (-1)^{1/3} \sqrt{3} \right. \right. \\
& \quad \left. \left. (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}, (-1)^{1/3} \right] \right] \right) \right) / \\
& \quad \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right) \Big)
\end{aligned}$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \quad \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}, (-1)^{1/3} \right] + (-1)^{1/3} \sqrt{3} \right. \right. \\
& \quad \left. \left. (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}, (-1)^{1/3} \right] \right] \right) \right) / \\
& \quad \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3} x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \frac{1}{2\sqrt{2}} 3^{1/4} \left((i + \sqrt{3}) a^{1/3} - (-i + \sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + 3i \right. \\
& \left. a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right) / \\
& \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 dx}{(c + dx) \sqrt{c^3 - 8d^3 x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{(c-2dx)^2}{3\sqrt{c} \sqrt{c^3-8d^3x^3}} \right]}{3\sqrt{c} d}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{c - 2 dx}{(1 + (-1)^{1/3}) c}} \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} c + 2 dx \right) \sqrt{\frac{(-1)^{1/3} (c + 2 (-1)^{1/3} dx)}{(1 + (-1)^{1/3}) c}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \right], (-1)^{1/3} \right] + \right. \right. \\
& \left. (-1)^{1/3} \sqrt{3} (1 + (-1)^{1/3}) c \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{\frac{c^2 + 2c dx + 4d^2 x^2}{c^2}} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \right], (-1)^{1/3} \right] \right) \right) / \\
& \left((-2 + (-1)^{1/3}) d \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8d^3 x^3} \right)
\end{aligned}$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 - x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 139 leaves, 4 steps):

$$\frac{2}{9} (e + 2 f) \operatorname{ArcTanh}\left[\frac{(1+x)^2}{3\sqrt{1+x^3}}\right] + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}-x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 273 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{-i(1+x)}{-3i+\sqrt{3}}} \right. \\ \left. \left(-3if\sqrt{i+\sqrt{3}}-2ix(-i-\sqrt{3}+(-i+\sqrt{3})x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3}(e+2f)\sqrt{-i+\sqrt{3}+2ix} \right. \right. \\ \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right) \right) / \left((3i+\sqrt{3})\sqrt{-i+\sqrt{3}+2ix}\sqrt{1+x^3} \right)$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 + x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$-\frac{2}{9} (e - 2 f) \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right] - \frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 271 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \right. \\ \left. \left(3f \sqrt{i+\sqrt{3}+2ix} (-1+i\sqrt{3}+x+i\sqrt{3}x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] - 2\sqrt{3}(e-2f) \sqrt{-i+\sqrt{3}-2ix} \right. \right. \\ \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right) \right) / \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{1-x^3} \right)$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$-\frac{2}{9}(e-2f) \operatorname{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right] - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 269 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \right. \\ \left(3f \sqrt{i+\sqrt{3}+2ix} (-1+i\sqrt{3}+x+i\sqrt{3}x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] - 2\sqrt{3}(e-2f) \sqrt{-i+\sqrt{3}-2ix} \right. \\ \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right) \right) / \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{-1+x^3} \right)$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 150 leaves, 4 steps):

$$\frac{2}{9} (e + 2 f) \operatorname{ArcTan}\left[\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right] + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 275 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{-i(1+x)}{-3i+\sqrt{3}}} \right. \\ \left. \left(-3if\sqrt{i+\sqrt{3}-2ix}(-i-\sqrt{3}+(-i+\sqrt{3})x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 2\sqrt{3}(e+2f)\sqrt{-i+\sqrt{3}+2ix} \right. \right. \\ \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right] \right) \Big/ \left((3i+\sqrt{3})\sqrt{-i+\sqrt{3}+2ix}\sqrt{-1-x^3} \right)$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(2a^{1/3}-b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{2(b^{1/3}e+2a^{1/3}f)\operatorname{ArcTanh}\left[\frac{(a^{1/3}+b^{1/3}x)^2}{3a^{1/6}\sqrt{a+bx^3}}\right]}{9\sqrt{a}b^{2/3}} + \\ \left(2\sqrt{2+\sqrt{3}}(b^{1/3}e-a^{1/3}f)(a^{1/3}+b^{1/3}x)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) \Big/ \\ \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 419 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \frac{1}{2\sqrt{2}} 3^{1/4} f \left((i + \sqrt{3}) a^{1/3} - (-i + \sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + \right. \\
& \left. i (b^{1/3} e + 2 a^{1/3} f) \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right) / \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 304 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 (b^{1/3} e - 2 a^{1/3} f) \operatorname{ArcTanh} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{9 \sqrt{a} b^{2/3}} \\
& \left(2 \sqrt{2 + \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 447 leaves):

$$\frac{1}{(-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2} i f \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \left((-3i + \sqrt{3}) a^{1/3} - (3i + \sqrt{3}) b^{1/3} x \right) \right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] - i(b^{1/3}e - 2a^{1/3}f) \sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}$$

$$\left. \sqrt{1 + \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 313 leaves, 4 steps):

$$\frac{2 (b^{1/3} e - 2 a^{1/3} f) \text{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}}\right]}{9 \sqrt{a} b^{2/3}}$$

$$\left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4 \sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)$$

Result (type 4, 448 leaves):

$$\frac{1}{(-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2} i f \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \left((-3i + \sqrt{3}) a^{1/3} - (3i + \sqrt{3}) b^{1/3} x \right) \right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] - i(b^{1/3}e - 2a^{1/3}f) \sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}$$

$$\left. \sqrt{1 + \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 310 leaves, 4 steps):

$$\frac{2 (b^{1/3} e + 2 a^{1/3} f) \text{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}}\right]}{9 \sqrt{a} b^{2/3}} +$$

$$\left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e - a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4 \sqrt{3}\right] \right) /$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)$$

Result (type 4, 422 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \frac{1}{2\sqrt{2}} 3^{1/4} f \left((i + \sqrt{3}) a^{1/3} - (-i + \sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2 i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] + \right. \\
& \left. i (b^{1/3} e + 2 a^{1/3} f) \sqrt{\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] \right) / \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 4, 221 leaves, 4 steps):

$$\frac{2 (d e - c f) \operatorname{ArcTanh} \left[\frac{(c - 2 d x)^2}{3 \sqrt{c} \sqrt{c^3 - 8 d^3 x^3}} \right] - \sqrt{2 + \sqrt{3}} (2 d e + c f) (c - 2 d x) \sqrt{\frac{c^2 + 2 c d x + 4 d^2 x^2}{((1 + \sqrt{3}) c - 2 d x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) c - 2 d x}{(1 + \sqrt{3}) c - 2 d x} \right], -7 - 4 \sqrt{3} \right]}{9 c^{3/2} d^2 - 3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c - 2 d x)}{((1 + \sqrt{3}) c - 2 d x)^2}} \sqrt{c^3 - 8 d^3 x^3}}$$

Result (type 4, 384 leaves):

$$\begin{aligned}
& - \left(\left(i \sqrt{\frac{c-2dx}{(1+(-1)^{1/3})c}} \right. \right. \\
& \left. \left(f \sqrt{\frac{(-i+\sqrt{3})c+2(i+\sqrt{3})dx}{(-3i+\sqrt{3})c}} \left((-3i+\sqrt{3})c-2(3i+\sqrt{3})dx \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{2} \sqrt{\frac{ic+idx+\sqrt{3}dx}{3ic-\sqrt{3}c}} \right], \frac{1}{2}(1+i\sqrt{3}) \right] + \right. \right. \\
& \left. \left. 4\sqrt{2}(de-cf) \sqrt{\frac{ic+idx+\sqrt{3}dx}{3ic-\sqrt{3}c}} \sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{2} \sqrt{\frac{ic+idx+\sqrt{3}dx}{3ic-\sqrt{3}c}} \right], \frac{1}{2}(1+i\sqrt{3}) \right] \right) \right) / \left(2(-2+(-1)^{1/3})d^2 \sqrt{\frac{c-2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}} \sqrt{c^3-8d^3x^3} \right)
\end{aligned}$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$\frac{4}{9} \text{ArcTanh} \left[\frac{(1+x)^2}{3\sqrt{1+x^3}} \right] - \frac{2\sqrt{2+\sqrt{3}}(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}}$$

$$\left(\frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1+(-1)^{1/3}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{2i\sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{-2+(-1)^{1/3}} \right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right] - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}}$$

$$\left(\frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2i\sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 148 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right] - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}}$$

$$\left(\frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2 i \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 140 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTan}\left[\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right] - \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}}$$

$$\left(\frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2 i \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2a^{1/3} - b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTanh}\left[\frac{(a^{1/3}+b^{1/3}x)^2}{3a^{1/6}\sqrt{a+bx^3}}\right]}{9a^{1/6}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}(a^{1/3}+b^{1/3}x)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3}}$$

Result (type 4, 407 leaves):

$$\left(\sqrt{\frac{a^{1/3}+b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}} \left(-\sqrt{2} 3^{1/4} \left((i+\sqrt{3})a^{1/3} - (-i+\sqrt{3})b^{1/3}x \right) \sqrt{i+\sqrt{3} - \frac{2ib^{1/3}x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{-2ia^{1/3}+(i+\sqrt{3})b^{1/3}x}{(-3i+\sqrt{3})a^{1/3}}\right], \frac{1}{2}(1+i\sqrt{3})\right] + 8i \right. \right. \\ \left. \left. a^{1/3} \sqrt{\frac{-2ia^{1/3}+(i+\sqrt{3})b^{1/3}x}{(-3i+\sqrt{3})a^{1/3}}} \sqrt{1 - \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{-2ia^{1/3}+(i+\sqrt{3})b^{1/3}x}{(-3i+\sqrt{3})a^{1/3}}\right], \frac{1}{2}(1+i\sqrt{3})\right] \right) \right) / \\ \left(2(-2+(-1)^{1/3})b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}} \sqrt{a+bx^3} \right)$$

Problem 97: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2a^{1/3}+b^{1/3}x)\sqrt{a-bx^3}} dx$$

Optimal (type 4, 268 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTanh}\left[\frac{(a^{1/3}-b^{1/3}x)^2}{3a^{1/6}\sqrt{a-bx^3}}\right]}{9a^{1/6}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}(a^{1/3}-b^{1/3}x)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}-b^{1/3}x}{(1+\sqrt{3})a^{1/3}-b^{1/3}x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3}-b^{1/3}x)}{((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \sqrt{a-bx^3}}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right. \\
& \left. \frac{1}{\sqrt{3}} 2 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \left. \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTan} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3}}$$

Result (type 4, 372 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \frac{1}{\sqrt{3}} 2 (-1)^{1/3} \right. \\
& \left. \left. \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \\
& \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 273 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTan} \left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3}}$$

Result (type 4, 410 leaves):

$$\left(\sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
\left. \left(-\sqrt{2} 3^{1/4} \left((i + \sqrt{3}) a^{1/3} - (-i + \sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] + 8i \right. \right. \\
\left. \left. a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] \right) \right) / \\
\left(2 (-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + dx) \sqrt{c^3 - 8d^3 x^3}} dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}} \right]}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx) \sqrt{\frac{-c^2+2cdx+4d^2x^2}{((1+\sqrt{3})c-2dx)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1-\sqrt{3})c-2dx}{(1+\sqrt{3})c-2dx} \right], -7-4\sqrt{3} \right]}{3 \times 3^{1/4} d^2 \sqrt{\frac{c(c-2dx)}{((1+\sqrt{3})c-2dx)^2}} \sqrt{c^3-8d^3x^3}}$$

Result (type 4, 295 leaves):

$$\left(\sqrt{\frac{c-2dx}{(1+(-1)^{1/3})c}} \left((-2+(-1)^{1/3}) \left((-1)^{1/3}c+2dx \right) \sqrt{\frac{(-1)^{1/3}(c+2(-1)^{1/3}dx)}{(1+(-1)^{1/3})c}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c-2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}}\right], (-1)^{1/3}\right] + \frac{1}{\sqrt{3}} \right. \right. \\ \left. \left. 2(-1)^{1/3}(1+(-1)^{1/3})c \sqrt{\frac{c-2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}} \sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{c-2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}}\right], (-1)^{1/3}\right] \right) \right) / \\ \left((-2+(-1)^{1/3})d^2 \sqrt{\frac{c-2(-1)^{2/3}dx}{(1+(-1)^{1/3})c}} \sqrt{c^3-8d^3x^3} \right)$$

Problem 101: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{-3+2\sqrt{3}}}$$

Result (type 4, 267 leaves):

$$-\left(\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \right. \\ \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left((1+2i) - i\sqrt{3} + ((-2-i)+\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 4i\sqrt{i+\sqrt{3}-2ix} \right. \right. \\ \left. \left. \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((-3+(2+i)\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3} \right) \right)$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{-3+2\sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2ix} \left((2+i) - \sqrt{3} + ((1+2i) - i\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 4\sqrt{-i+\sqrt{3}-2ix} \sqrt{1+x+x^2} \right. \right. \\ \left. \left. \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \left((-3i+(1+2i)\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{1-x^3} \right)$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{-3+2\sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2ix} \left((2+i) - \sqrt{3} + ((1+2i) - i\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] + 4\sqrt{-i+\sqrt{3}-2ix} \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \left((-3i+(1+2i)\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{-1+x^3} \right)$$

Problem 104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{-3+2\sqrt{3}}}$$

Result (type 4, 269 leaves):

$$-\left(\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \left(\sqrt{-i+\sqrt{3}+2ix} \left((1+2i) - i\sqrt{3} + ((-2-i) + \sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 4i\sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((-3+(2+i)\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{-1-x^3} \right) \right)$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3}x)}{\sqrt{a+bx^3}}\right]}{\sqrt{-3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 322 leaves):

$$\frac{1}{\sqrt{a + b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{((-1)^{1/3} a^{1/3} - b^{1/3} x) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \left(4 (-1)^{5/6} (1 + (-1)^{1/3}) a^{1/3} \right. \right.$$

$$\left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left((-3i + (1+2i)\sqrt{3}) b^{1/3} \right) \right)$$

Problem 106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3}x)}{\sqrt{a-bx^3}}\right]}{\sqrt{-3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 446 leaves):

$$\frac{1}{(-3i + (1+2i)\sqrt{3}) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \sqrt{a-bx^3}} + 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \left(\sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \left((-3 + (2+i)\sqrt{3}) a^{1/3} + (-3i + (1+2i)\sqrt{3}) b^{1/3} x \right) \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1-i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1+i\sqrt{3})\right] + 4\sqrt{3} a^{1/3} \sqrt{-\frac{2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right. \right. \\ \left. \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1-i\sqrt{3})b^{1/3}x)}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1+i\sqrt{3})\right]\right) \right)$$

Problem 107: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3}x)}{\sqrt{-a+bx^3}}\right]}{\sqrt{-3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 447 leaves):

$$\frac{1}{(-3i + (1 + 2i)\sqrt{3}) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{-a + b x^3}}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \left(\sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}} \left((-3 + (2 + i)\sqrt{3}) a^{1/3} + (-3i + (1 + 2i)\sqrt{3}) b^{1/3} x \right)} \right.}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3})b^{1/3}x}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] + 4\sqrt{3} a^{1/3} \sqrt{-\frac{2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}$$

$$\left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3})b^{1/3}x}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)$$

Problem 108: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}}\right]}{\sqrt{-3 + 2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 325 leaves):

$$\frac{1}{\sqrt{-a - b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \left(4 (-1)^{5/6} (1 + (-1)^{1/3}) a^{1/3} \right. \right.$$

$$\left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 i + (1 + 2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left((-3 i + (1 + 2 i) \sqrt{3}) b^{1/3} \right) \right)$$

Problem 109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+bx^3}}\right]}{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 862 leaves):

$$\begin{aligned}
& \frac{1}{3(-5+3\sqrt{3})(2(-5+3\sqrt{3})a-bx^3)\sqrt{a+bx^3}} (26-15\sqrt{3})ax \\
& \left(- \left(\left(96(-1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(8(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(\left(60(-3+\sqrt{3})a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \right. \\
& \quad \left(10(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(- \left(\left(16\sqrt{3}a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(4(-5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. bx^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) - \\
& \quad \left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(14(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 110: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x)\sqrt{a-bx^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\left(\frac{b}{a}\right)^{1/3}x\right)}{\sqrt{a-bx^3}}\right]}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3\sqrt{3} \right) \sqrt{-a + bx^3} \left(2 \left(-5 + 3\sqrt{3} \right) a + bx^3 \right)} \left(26 - 15\sqrt{3} \right) a x \\
& \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) / \left(8 \left(-5 + 3\sqrt{3} \right) a \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 3bx^3 \left(\operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] + \left(5 - 3\sqrt{3} \right) \operatorname{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) \right) \right) \right) + \\
& \quad \times \left(- \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) / \left(10 \left(-5 + 3\sqrt{3} \right) a \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 3bx^3 \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] + \left(5 - 3\sqrt{3} \right) \operatorname{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) \right) \right) \right) + \\
& \quad \times \left(- \left(\left(16\sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \operatorname{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) / \left(4 \left(-5 + 3\sqrt{3} \right) a \operatorname{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. bx^3 \left(\operatorname{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] + \left(5 - 3\sqrt{3} \right) \operatorname{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) \right) \right) \right) + \\
& \quad \left(21bx \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) / \left(14 \left(-5 + 3\sqrt{3} \right) a \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] - \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] + \left(5 - 3\sqrt{3} \right) \operatorname{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a - 6\sqrt{3}a} \right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 111: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x}{\left(1 - \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x \right) \sqrt{-a + bx^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a} \right)^{1/3} x \right)}{\sqrt{-a+bx^3}} \right]}{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a} \right)^{1/3}}$$

Result (type 6, 836 leaves):

$$\begin{aligned}
& \frac{1}{3(-5+3\sqrt{3})\sqrt{-a+bx^3}} (26-15\sqrt{3})ax \\
& \left(- \left(\left(96(-1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) / \left(8(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) - \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(- \left(\left(60(-3+\sqrt{3})a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) / \left(10(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) - \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(- \left(\left(16\sqrt{3}a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) / \left(4(-5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) - \right. \right. \\
& \quad \left. \left. bx^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) / \left(14(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] - \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 112: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{(1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x) \sqrt{-a - bx^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a-bx^3}}\right]}{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 865 leaves):

$$\begin{aligned}
& \frac{1}{3(-5+3\sqrt{3})\sqrt{-a-bx^3}} \left((26-15\sqrt{3})ax \right. \\
& \left. - \left(\left(96(-1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(8(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(\left(60(-3+\sqrt{3})a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \right. \\
& \quad \left(10(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) + \\
& \quad \times \left(- \left(\left(16\sqrt{3}a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(4(-5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. bx^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) - \\
& \quad \left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{-10a+6\sqrt{3}a}\right] \right) / \left(14(-5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] + (5-3\sqrt{3}) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a-6\sqrt{3}a}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{1+x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3+2\sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left((-2-i) - \sqrt{3} + ((1+2i) + i\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 4\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \right. \right. \\ \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((3i+(1+2i)\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3} \right)$$

Problem 114: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x)\sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{3+2\sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \right. \\ \left(\sqrt{i+\sqrt{3}+2ix} \left((1+2i) + i\sqrt{3} + ((2+i) + \sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] - 4i\sqrt{-i+\sqrt{3}-2ix} \right. \\ \left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \left((3+(2+i)\sqrt{3})\sqrt{-i+\sqrt{3}-2ix}\sqrt{1-x^3} \right)$$

Problem 115: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{3+2\sqrt{3}}}$$

Result (type 4, 265 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2ix} \left((1+2i) + i\sqrt{3} + ((2+i)+\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] - 4i\sqrt{-i+\sqrt{3}-2ix} \right. \right. \right. \\ \left. \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right]\right] \right) \right) / \left((3+(2+i)\sqrt{3})\sqrt{-i+\sqrt{3}-2ix}\sqrt{-1+x^3} \right)$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3+2\sqrt{3}}}$$

Result (type 4, 271 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left((-2-i) - \sqrt{3} + ((1+2i) + i\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 4\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2} \right. \right. \\ \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((3i+(1+2i)\sqrt{3})\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3} \right)$$

Problem 117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3}x)}{\sqrt{a+bx^3}}\right]}{\sqrt{3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 320 leaves):

$$\frac{1}{\sqrt{a+bx^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}} \left(- \frac{((-1)^{1/3}a^{1/3} - b^{1/3}x) \sqrt{(-1)^{1/6} - \frac{ib^{1/3}x}{a^{1/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3}b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}}\right], (-1)^{1/3}\right]}{3^{1/4}b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3}b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}}} + \frac{1}{(3+(2+i)\sqrt{3})b^{1/3}} \right. \\ \left. 4(-1)^{1/3}(1+(-1)^{1/3})a^{1/3} \sqrt{1 - \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3}b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{\sqrt{3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 329 leaves):

$$\frac{1}{b^{1/3} \sqrt{a - b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{((-1)^{1/3} a^{1/3} + b^{1/3} x) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}}}{\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] - \frac{1}{3 + (2 + i) \sqrt{3}} \right.$$

$$\left. 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3}x)}{\sqrt{-a+bx^3}}\right]}{\sqrt{3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 330 leaves):

$$\frac{1}{b^{1/3} \sqrt{-a+bx^3}}$$

$$2 \sqrt{\frac{a^{1/3}-b^{1/3}x}{(1+(-1)^{1/3})a^{1/3}}}$$

$$\left(\frac{\left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1+(-1)^{1/3})a^{1/3}}}}{\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3})a^{1/3}}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3})a^{1/3}}}\right], (-1)^{1/3}\right]}{3 + (2+i)\sqrt{3}} - \frac{1}{3 + (2+i)\sqrt{3}} \right.$$

$$\left. 4 (-1)^{1/3} (1+(-1)^{1/3}) a^{1/3} \sqrt{1 + \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2i\sqrt{3}}{3 + (2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3})a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{-a-bx^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3}x)}{\sqrt{-a-bx^3}}\right]}{\sqrt{3+2\sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 323 leaves):

$$\frac{1}{\sqrt{-a - b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(3 + (2 + i) \sqrt{3}) b^{1/3}} \right.$$

$$\left. 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 121: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+bx^3}}\right]}{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 866 leaves):

$$\frac{1}{3(5+3\sqrt{3})\sqrt{a+bx^3}(2(5+3\sqrt{3})a+bx^3)}(26+15\sqrt{3})ax$$

$$\left(-\left(\left(96(1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(8(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]-\right.\right.\right.$$

$$\left.\left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)+x$$

$$\left(\left(60(3+\sqrt{3})a\left(\frac{b}{a}\right)^{1/3}\operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(10(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]-\right.\right.$$

$$\left.\left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)+$$

$$x\left(-\left(\left(16\sqrt{3}a\left(\frac{b}{a}\right)^{2/3}\operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(4(5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]-\right.\right.$$

$$\left.\left.\left. bx^3\left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)+$$

$$\left(\left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(14(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]-\right.\right.$$

$$\left.\left.\left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)\right)$$

Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x)\sqrt{a-bx^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\left(\frac{b}{a}\right)^{1/3}x\right)}{\sqrt{a-bx^3}}\right]}{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3(5+3\sqrt{3})\sqrt{a-bx^3}(2(5+3\sqrt{3})a-bx^3)} (26+15\sqrt{3})ax \\
& \left(- \left(\left(96(1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(8(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) \right) + \\
& \quad \times \left(- \left(\left(60(3+\sqrt{3})a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(10(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) \right) + \\
& \quad \times \left(- \left(\left(16\sqrt{3}a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(4(5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) + \right. \right. \\
& \quad \left. \left. bx^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) \right) - \\
& \quad \left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(14(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] + \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{bx^3}{a}, \frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{(1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x) \sqrt{-a + bx^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a+bx^3}}\right]}{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 836 leaves):

$$\begin{aligned}
& \frac{1}{3(5+3\sqrt{3})\left(2(5+3\sqrt{3})a-bx^3\right)\sqrt{-a+bx^3}}(26+15\sqrt{3})ax \\
& \left(-\left(\left(96(1+\sqrt{3})a\operatorname{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(8(5+3\sqrt{3})a\operatorname{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)+\right.\right. \\
& \quad \left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)+ \\
& \quad \times\left(-\left(\left(60(3+\sqrt{3})a\left(\frac{b}{a}\right)^{1/3}\operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(10(5+3\sqrt{3})a\operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)+\right.\right. \\
& \quad \left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)+ \\
& \quad \times\left(-\left(\left(16\sqrt{3}a\left(\frac{b}{a}\right)^{2/3}\operatorname{AppellF1}\left[1,\frac{1}{2},1,2,\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(4(5+3\sqrt{3})a\operatorname{AppellF1}\left[1,\frac{1}{2},1,2,\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)+\right.\right. \\
& \quad \left.\left. bx^3\left(\operatorname{AppellF1}\left[2,\frac{1}{2},2,3,\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[2,\frac{3}{2},1,3,\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)- \\
& \quad \left(\left(21bx\operatorname{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)/\left(14(5+3\sqrt{3})a\operatorname{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)+\right. \\
& \quad \left.\left.3bx^3\left(\operatorname{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\operatorname{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)
\end{aligned}$$

Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{-a-bx^3}}dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$-\frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\left(\frac{b}{a}\right)^{1/3}x\right)}{\sqrt{-a-bx^3}}\right]}{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 869 leaves):

$$\begin{aligned}
& \frac{1}{3(5+3\sqrt{3})\sqrt{-a-bx^3}(2(5+3\sqrt{3})a+bx^3)} (26+15\sqrt{3})ax \\
& \left(- \left(\left(96(1+\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(8(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) - \right. \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) + x \\
& \left(\left(60(3+\sqrt{3})a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(10(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) - \right. \\
& \quad \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) + \\
& x \left(- \left(\left(16\sqrt{3}a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(4(5+3\sqrt{3})a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) - \right. \right. \\
& \quad \left. \left. bx^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) + \\
& \left(21bx \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) / \left(14(5+3\sqrt{3})a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] - \right. \\
& \quad \left. \left. 3bx^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] + (5+3\sqrt{3}) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{bx^3}{10a+6\sqrt{3}a}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}}$$

Result (type 4, 269 leaves):

$$\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \\ \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left((-2-i) - \sqrt{3} + ((1+2i) + i\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 2\sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \right. \right. \\ \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((3i+(1+2i)\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{1+x^3} \right)$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}}$$

Result (type 4, 267 leaves):

$$-\left(\left(2\sqrt{6} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \right. \right. \\ \left. \left(\sqrt{-i+\sqrt{3}+2ix} \left((1+2i) - i\sqrt{3} + ((-2-i) + \sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + 2i\sqrt{i+\sqrt{3}-2ix} \right. \right. \\ \left. \left. \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \left((-3+(2+i)\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{1+x^3} \right)$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$\frac{(e - f - \sqrt{3} f) \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right] + \sqrt{2+\sqrt{3}} (e - (1-\sqrt{3}) f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})} + 3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 291 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \left(3f \sqrt{-i+\sqrt{3}+2ix} \left((-2-i) - \sqrt{3} + ((1+2i) + i\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 2(-\sqrt{3}e + (3+\sqrt{3})f) \sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \\ \left((3i+(1+2i)\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{1+x^3} \right)$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 187 leaves, 4 steps):

$$\frac{(e+f+\sqrt{3}f) \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right] + \sqrt{2+\sqrt{3}} (e+(1-\sqrt{3})f) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})} + 3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 291 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \left(-3if\sqrt{-i+\sqrt{3}-2ix} \left(-i((2+i)+\sqrt{3}) + ((2-i)+\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 2(\sqrt{3}e + (3+\sqrt{3})f)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right) \right) / \\ \left((3i+(1+2i)\sqrt{3})\sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3} \right)$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 190 leaves, 4 steps):

$$\frac{(e+f+\sqrt{3}f)\text{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right] - \sqrt{2-\sqrt{3}}(e+(1-\sqrt{3})f)(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})} \cdot 3^{3/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 289 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \left(-3if\sqrt{-i+\sqrt{3}-2ix} \left(-i((2+i)+\sqrt{3}) + ((2-i)+\sqrt{3})x \right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 2(\sqrt{3}e + (3+\sqrt{3})f)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right) \right) / \\ \left((3i+(1+2i)\sqrt{3})\sqrt{i+\sqrt{3}+2ix}\sqrt{-1+x^3} \right)$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 183 leaves, 4 steps):

$$\frac{(e - (1 + \sqrt{3}) f) \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right] + \sqrt{2-\sqrt{3}} (e - (1 - \sqrt{3}) f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{\sqrt{3(3+2\sqrt{3})} \cdot 3^{3/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 293 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{i(1+x)}{3i+\sqrt{3}}} \left(3 f \sqrt{-i+\sqrt{3}+2ix} \left((-2-i) - \sqrt{3} + ((1+2i) + i\sqrt{3}) x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] + \right. \right. \\ \left. \left. 2(-\sqrt{3} e + (3+\sqrt{3}) f) \sqrt{i+\sqrt{3}-2ix} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \\ \left((3i+(1+2i)\sqrt{3}) \sqrt{i+\sqrt{3}-2ix} \sqrt{-1-x^3} \right)$$

Problem 131: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 332 leaves, 4 steps):

$$\frac{(b^{1/3} e - (1 - \sqrt{3}) a^{1/3} f) \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{a + b x^3}}\right]}{\sqrt{3(-3+2\sqrt{3})} \sqrt{a} b^{2/3}} \\ \left(\sqrt{2+\sqrt{3}} (b^{1/3} e - (1 + \sqrt{3}) a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4\sqrt{3}\right] \right) / \\ \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 438 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} i 3^{1/4} f \left(((-2 - i) + \sqrt{3}) a^{1/3} + ((1 + 2i) - i\sqrt{3}) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \right. \\
& \quad \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + i (b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f) \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \\
& \quad \left. \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right) \right) \right) / \\
& \quad \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 336 leaves, 4 steps):

$$\begin{aligned}
& \frac{(b^{1/3} e + (1 - \sqrt{3}) a^{1/3} f) \text{ArcTanh} \left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{\sqrt{3(-3 + 2\sqrt{3})} \sqrt{a} b^{2/3}} + \\
& \left(\sqrt{2 + \sqrt{3}} (b^{1/3} e + (1 + \sqrt{3}) a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 466 leaves):

$$\begin{aligned}
& - \frac{1}{(3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{a - b x^3}}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} f (i(-3 + (2 + i)\sqrt{3}) a^{1/3} + (3 - (2 - i)\sqrt{3}) b^{1/3} x) \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] - i(b^{1/3} e - (-1 + \sqrt{3}) a^{1/3} f) \sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] \right)
\end{aligned}$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps):

$$\frac{(b^{1/3} e + (1 - \sqrt{3}) a^{1/3} f) \text{ArcTan}\left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{-a + b x^3}}\right]}{\sqrt{3(-3 + 2\sqrt{3})} \sqrt{a} b^{2/3}} +$$

$$\left(\sqrt{2 - \sqrt{3}} (b^{1/3} e + (1 + \sqrt{3}) a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4\sqrt{3}\right] \right) /$$

$$\left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)$$

Result (type 4, 467 leaves):

$$\begin{aligned}
& - \frac{1}{(3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{-a + b x^3}}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} f (i(-3 + (2 + i)\sqrt{3}) a^{1/3} + (3 - (2 - i)\sqrt{3}) b^{1/3} x) \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] - i(b^{1/3} e - (-1 + \sqrt{3}) a^{1/3} f) \sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)
\end{aligned}$$

Problem 134: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps):

$$\frac{(b^{1/3} e - (1 - \sqrt{3}) a^{1/3} f) \text{ArcTan}\left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}}\right]}{\sqrt{3(-3 + 2\sqrt{3})} \sqrt{a} b^{2/3}}$$

$$\left(\sqrt{2 - \sqrt{3}} (b^{1/3} e - (1 + \sqrt{3}) a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4\sqrt{3}\right] \right) / \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)$$

Result (type 4, 441 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} i 3^{1/4} f \left(\left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2i) - i\sqrt{3} \right) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \right. \\
& \quad \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + i (b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f) \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \\
& \quad \left. \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right] \right) \right) / \\
& \quad \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 136 leaves, 4 steps):

$$\frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}} \right]}{3^{3/4}} + \frac{\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right]}{3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 209 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{2 i \left(1 + \sqrt{3} \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3+(2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{3+(2+i)\sqrt{3}} \right)$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1+\sqrt{3}-x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 152 leaves, 4 steps):

$$- \frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right]}{3^{3/4}} + \frac{\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 232 leaves):

$$\frac{1}{(3+(2+i)\sqrt{3})\sqrt{1-x^3}} \\ 2 i \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} i \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \left(3 i + (1+2 i) \sqrt{3} + (3+(2+i)\sqrt{3}) x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\ \left. 2 \left(1 + \sqrt{3} \right) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3+(2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right]}{3^{3/4}} + \frac{2 \sqrt{\frac{7}{6} - \frac{2}{\sqrt{3}}}(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 230 leaves):

$$\left(2i \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} i \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} (3i + (1+2i)\sqrt{3} + (3+(2+i)\sqrt{3})x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \right. \\ \left. \left. 2(1+\sqrt{3}) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \right) / \left((3+(2+i)\sqrt{3}) \sqrt{-1+x^3} \right)$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right]}{3^{3/4}} + \frac{2 \sqrt{\frac{7}{6} - \frac{2}{\sqrt{3}}}(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 211 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{2i(1+\sqrt{3})\sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{3+(2+i)\sqrt{3}} \right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}}$$

Result (type 4, 225 leaves):

$$\left(2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \left(3 - (2+i)\sqrt{3} + (-3i + (1+2i)\sqrt{3})x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] - \right. \right. \\ \left. \left. 2(-1+\sqrt{3})\sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \right) / \left((-3i + (1+2i)\sqrt{3})\sqrt{1+x^3} \right)$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3}x)}{\sqrt{a+bx^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4\sqrt{3}\right]}{3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}}$$

Result (type 4, 427 leaves):

$$\begin{aligned} & - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} i 3^{1/4} \left(\left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2i) - i\sqrt{3} \right) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \right. \\ & \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + i\sqrt{3})\right] + i (-1 + \sqrt{3}) a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \right. \\ & \quad \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + i\sqrt{3})\right] \right) \right) / \\ & \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right) \end{aligned}$$

Problem 141: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x \right) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3}x)}{\sqrt{a-bx^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (a^{1/3} - b^{1/3}x) \sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}-b^{1/3}x}{(1+\sqrt{3})a^{1/3}-b^{1/3}x}\right], -7-4\sqrt{3}\right]}{3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3}-b^{1/3}x)}{((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \sqrt{a-bx^3}}$$

Result (type 4, 454 leaves):

$$\frac{1}{(3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3}}$$

$$4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} (i(-3 + (2 + i)\sqrt{3}) a^{1/3} + (3 - (2 - i)\sqrt{3}) b^{1/3} x) \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}} \right.$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3})b^{1/3}x}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] + i(-1 + \sqrt{3}) a^{1/3} \sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3})b^{1/3}x}{(-3i + \sqrt{3})a^{1/3}}}$$

$$\left. \sqrt{1 + \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3})b^{1/3}x}{(-3i + \sqrt{3})a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)$$

Problem 142: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3}x)}{\sqrt{-a+bx^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{\sqrt{2} (a^{1/3} - b^{1/3}x) \sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1-\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1+\sqrt{3})a^{1/3}-b^{1/3}x}{(1-\sqrt{3})a^{1/3}-b^{1/3}x}\right], -7+4\sqrt{3}\right]}{3^{3/4} b^{2/3} \sqrt{-\frac{a^{1/3}(a^{1/3}-b^{1/3}x)}{((1-\sqrt{3})a^{1/3}-b^{1/3}x)^2}} \sqrt{-a+bx^3}}$$

Result (type 4, 455 leaves):

$$\begin{aligned}
& - \frac{1}{(3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{-a + b x^3}}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} (i(-3 + (2 + i)\sqrt{3}) a^{1/3} + (3 - (2 - i)\sqrt{3}) b^{1/3} x) \sqrt{\frac{(-i + \sqrt{3}) a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right] + i(-1 + \sqrt{3}) a^{1/3} \sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{i(2a^{1/3} + (1 - i)\sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2}(1 + i\sqrt{3})\right]\right)
\end{aligned}$$

Problem 143: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned}
& \frac{\sqrt{2} \text{ArcTan}\left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}}\right] + \sqrt{2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4\sqrt{3}\right]}{3^{3/4} a^{1/6} b^{2/3}} \\
& + \frac{3^{3/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3}}{3^{3/4} a^{1/6} b^{2/3}}
\end{aligned}$$

Result (type 4, 430 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} i 3^{1/4} \left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2i) - i\sqrt{3} \right) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \quad \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + i (-1 + \sqrt{3}) a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \\
& \quad \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right) / \\
& \quad \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 144: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{(c + dx) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 319 leaves, 6 steps):

$$\begin{aligned}
& \frac{(c - (1 + \sqrt{3})d) (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{ArcTan} \left[\frac{\sqrt{c^2 + cd + d^2} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}}{\sqrt{c - d} \sqrt{d} \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}} \right]}{\sqrt{c - d} \sqrt{d} \sqrt{c^2 + cd + d^2} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} \\
& \frac{4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticPi} \left[\frac{(c - (1 + \sqrt{3})d)^2}{(c - (1 - \sqrt{3})d)^2}, -\text{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4\sqrt{3} \right]}{(c - (1 - \sqrt{3})d) \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}
\end{aligned}$$

Result (type 4, 214 leaves):

$$\frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i \left(c - \left(1 + \sqrt{3} \right) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3} d} \right)$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{(c + dx) \sqrt{1-x^3}} dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$\frac{\left(c + d + \sqrt{3} d \right) \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3} - x \right)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{\left(1+\sqrt{3} - x \right)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3} - x \right)^2}}} \right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{\left(1+\sqrt{3} - x \right)^2}} \sqrt{1-x^3}} + \\ \frac{4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3} - x \right)^2}} \operatorname{EllipticPi}\left[\frac{\left(c+d+\sqrt{3} d \right)^2}{\left(c+d-\sqrt{3} d \right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7 - 4 \sqrt{3} \right]}{\left(c + d - \sqrt{3} d \right) \sqrt{\frac{1-x}{\left(1+\sqrt{3} - x \right)^2}} \sqrt{1-x^3}}$$

Result (type 4, 235 leaves):

$$\frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3} d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 146: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{(c + d x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 327 leaves, 6 steps):

$$\frac{(c + d + \sqrt{3} d) (1 - x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{ArcTanh} \left[\frac{\sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}} \right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}} + \\ \frac{4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticPi} \left[\frac{(c+d+\sqrt{3} d)^2}{(c+d-\sqrt{3} d)^2}, -\text{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7 - 4 \sqrt{3} \right]}{(c + d - \sqrt{3} d) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 233 leaves):

$$\frac{1}{3 d \sqrt{-1+x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3} d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{(c + d x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 323 leaves, 6 steps):

$$\frac{(c - (1 + \sqrt{3}) d) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan} \left[\frac{\sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{c-d} \sqrt{d} \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}} \right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}} \\ - \frac{4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi} \left[\frac{(c - (1 + \sqrt{3}) d)^2}{(c - (1 - \sqrt{3}) d)^2}, -\operatorname{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4 \sqrt{3} \right]}{(c - (1 - \sqrt{3}) d) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 216 leaves):

$$\frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i \left(c - \left(1 + \sqrt{3} \right) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3} d} \right)$$

Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{(c + dx) \sqrt{1+x^3}} dx$$

Optimal (type 4, 360 leaves, 6 steps):

$$\frac{\left(c - \left(1 - \sqrt{3} \right) d \right) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{ArcTanh}\left[\frac{2 \sqrt{2+\sqrt{3}} \sqrt{c^2+cd+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}}}{\sqrt{c-d} \sqrt{d} \sqrt{7+4\sqrt{3} + \frac{(1+\sqrt{3}+x)^2}{(1-\sqrt{3}+x)^2}}} \right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+cd+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\ \frac{4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c-(1-\sqrt{3})d)^2}{(c-(1+\sqrt{3})d)^2}, -\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right]}{(c-d-\sqrt{3}d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 213 leaves):

$$\frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i \left(c + (-1 + \sqrt{3}) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c + (-1)^{1/3} d} \right)$$

Problem 149: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{(c + dx) \sqrt{1-x^3}} dx$$

Optimal (type 4, 348 leaves, 6 steps):

$$\frac{(c + d - \sqrt{3} d) (1-x) \sqrt{\frac{1+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2 - cd + d^2} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x^2}{(1-\sqrt{3}-x)^2}}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - cd + d^2} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{1-x^3}} - \\ \frac{4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d-\sqrt{3}d)^2}{(c+d+\sqrt{3}d)^2}, -\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{(c+d+\sqrt{3}d) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 235 leaves):

$$\frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3} d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (-3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 150: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{(c + d x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\frac{(c + d - \sqrt{3} d) (1 - x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{ArcTan} \left[\frac{\sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{(1-\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}} \right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}} - \\ \frac{4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticPi} \left[\frac{(c+d-\sqrt{3} d)^2}{(c+d+\sqrt{3} d)^2}, -\text{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right]}{(c+d+\sqrt{3} d) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 233 leaves):

$$\frac{1}{3 d \sqrt{-1+x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3} d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (-3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{(c + d x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\frac{(c - (1 - \sqrt{3}) d) (1 + x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{ArcTanh} \left[\frac{2 \sqrt{2+\sqrt{3}} \sqrt{c^2+cd+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}}}{\sqrt{c-d} \sqrt{d} \sqrt{7+4\sqrt{3} + \frac{(1+\sqrt{3}+x)^2}{(1-\sqrt{3}+x)^2}}} \right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+cd+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \\ \frac{4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticPi} \left[\frac{(c-(1-\sqrt{3})d)^2}{(c-(1+\sqrt{3})d)^2}, -\text{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right]}{(c-d-\sqrt{3}d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 215 leaves):

$$\frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i \left(c + (-1 + \sqrt{3}) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c + (-1)^{1/3} d} \right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{1+x^3}} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$-\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] + \frac{2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 149 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] - \frac{2 \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right]}{\sqrt{3}} - \frac{2 \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x \right)}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1+x^3}}$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{1-x^3}} dx$$

Optimal (type 4, 139 leaves, 5 steps):

$$-\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1-x^3}] + \frac{2\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 157 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}[\sqrt{1-x^3}] - \frac{2 \operatorname{ArcTanh}[\sqrt{1-x^3}]}{\sqrt{3}} - \frac{2\sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \sqrt{1-x^3}}$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 142 leaves, 5 steps):

$$\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1+x^3}] + \frac{2\sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 150 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1+x^3}] + \sqrt{3} \operatorname{ArcTan}[\sqrt{-1+x^3}] - \frac{3\sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}} \right)$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 136 leaves, 5 steps):

$$\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 - x^3}] + \frac{2\sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}$$

Result (type 4, 155 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 - x^3}] + \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 - x^3}] - \frac{3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 - x^3}} \right)$$

Problem 156: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{1 + x^3}} dx$$

Optimal (type 4, 127 leaves, 5 steps):

$$-\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1 + x^3}] + \frac{2\sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}$$

Result (type 4, 149 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}[\sqrt{1 + x^3}] + \frac{2 \operatorname{ArcTanh}[\sqrt{1 + x^3}]}{\sqrt{3}} - \frac{2 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x \right)}{1 + (-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{1 + x^3}}$$

Problem 157: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal (type 4, 141 leaves, 5 steps):

$$-\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1 - x^3}] + \frac{2\sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x}\right], -7 - 4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3}}$$

Result (type 4, 158 leaves):

$$\frac{2}{3} \left(-\operatorname{ArcTanh}[\sqrt{1 - x^3}] + \sqrt{3} \operatorname{ArcTanh}[\sqrt{1 - x^3}] - \frac{3 \sqrt{\frac{1 - x}{1 + (-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{1 - x^3}} \right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 + x^3}] + \frac{2\sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x}\right], -7 + 4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3}}$$

Result (type 4, 151 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 + x^3}] - \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 + x^3}] - \frac{3 \sqrt{\frac{1 - x}{1 + (-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 + x^3}} \right)$$

Problem 159: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 138 leaves, 5 steps):

$$\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 - x^3}] + \frac{2\sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}$$

Result (type 4, 156 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 - x^3}] - \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 - x^3}] - \frac{3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 - x^3}} \right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3 + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 334 leaves, 8 steps):

$$\frac{3 (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}}{\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}}\right] - \frac{2\sqrt{2(97 + 56\sqrt{3})} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4\sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} - \frac{3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}{\sqrt{2 - \sqrt{3}} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} \operatorname{EllipticPi}\left[97 - 56\sqrt{3}, -\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4\sqrt{3}\right]}$$

Result (type 4, 194 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 i \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{3+(-1)^{1/3}} \right)$$

Problem 161: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 379 leaves, 8 steps):

$$\frac{3(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right] - 2 \sqrt{2(37+20\sqrt{3})} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{2\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} - 13 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

$$\frac{12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} (553+304\sqrt{3}), -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{1-x^3}} {}_2F_1 \left[\begin{matrix} 1-x \\ 1+(-1)^{1/3} \end{matrix} \right]$$

$$\left(\frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 i \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{5i+\sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{-3+(-1)^{1/3}} \right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 375 leaves, 8 steps):

$$\frac{3(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}} \right] + 2\sqrt{2}(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right]}{2\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3} - 3^{1/4} (4+\sqrt{3}) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

$$\frac{12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi} \left[\frac{1}{169} (553+304\sqrt{3}), -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{-1+x^3}} {}_2F_1 \left[\begin{matrix} 1-x \\ 1+(-1)^{1/3} \end{matrix} \right]$$

$$\left(\frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 i \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{5i+\sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{-3+(-1)^{1/3}} \right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 343 leaves, 8 steps):

$$\frac{3(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right] + 2\sqrt{14+8\sqrt{3}}(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3} - 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \frac{12 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56\sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 196 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{3i \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{3+(-1)^{1/3}} \right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}} dx$$

Optimal (type 4, 452 leaves, 8 steps):

$$\begin{aligned}
& \frac{(d e - c f) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{c-d} \sqrt{d} \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \frac{2 \sqrt{2+\sqrt{3}} (e - f - \sqrt{3} f) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} (c - d - \sqrt{3} d) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c - (1+\sqrt{3}) d)^2}{(c - (1-\sqrt{3}) d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\
& \left((c^2 - 2 c d - 2 d^2) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{f \left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
& \left. \frac{i (-d e + c f) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3} d} \right)
\end{aligned}$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 476 leaves, 8 steps):

$$\frac{(de - cf)(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 - cd + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{cd}} \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\right]}{\sqrt{d} \sqrt{cd} \sqrt{c^2 - cd + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}(e+f+\sqrt{3}f)(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3^{1/4}(c+d+\sqrt{3}d) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

$$+ \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}}(de - cf)(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d+\sqrt{3}d)^2}{(c+d-\sqrt{3}d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{(c^2 + 2cd - 2d^2) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 233 leaves):

$$\frac{1}{3d\sqrt{1-x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{3f \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{1}{-c + (-1)^{1/3}d} \right. \\ \left. (-1)^{1/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) (-de + cf) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}d}{-c + (-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + fx}{(c + dx) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 477 leaves, 8 steps):

$$\frac{(de - cf)(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 - cd + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{cd}} \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\right]}{\sqrt{d} \sqrt{cd} \sqrt{c^2 - cd + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(e+f+\sqrt{3}f)(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4}(c+d+\sqrt{3}d) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}} +$$

$$\frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}}(de - cf)(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d+\sqrt{3}d)^2}{(c+d-\sqrt{3}d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{(c^2 + 2cd - 2d^2) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 231 leaves):

$$\frac{1}{3d\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{3f((-1)^{1/3}+x) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{1}{-c+(-1)^{1/3}d} \right.$$

$$\left. (-1)^{1/3} \sqrt{3} (1+(-1)^{1/3}) (-de+cf) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}d}{-c+(-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 167: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 465 leaves, 8 steps):

$$\begin{aligned}
& \frac{(de - cf)(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+cd+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{c-d} \sqrt{d} \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+cd+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \\
& \frac{2\sqrt{2-\sqrt{3}}(e-f-\sqrt{3}f)(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3^{1/4}(c-d-\sqrt{3}d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (de - cf)(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c-(1+\sqrt{3})d)^2}{(c-(1-\sqrt{3})d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\
& \left((c^2 - 2cd - 2d^2) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{-1-x^3}} \left(2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{f((-1)^{1/3}-x) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \right. \right. \\
& \left. \left. \frac{i(-de+cf) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i\sqrt{3}d}{c+(-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3}d} \right) \right)
\end{aligned}$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{x \sqrt{1+x^3}} dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$-\frac{2}{3} e \operatorname{ArcTanh}[\sqrt{1+x^3}] + \frac{2\sqrt{2+\sqrt{3}} f(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 134 leaves):

$$-\frac{2}{3} e \operatorname{ArcTanh}[\sqrt{1+x^3}] - \frac{2 f \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x \right)}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1+x^3}}$$

Problem 169: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{1-x^3}} dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$-\frac{2}{3} e \operatorname{ArcTanh}[\sqrt{1-x^3}] - \frac{2\sqrt{2+\sqrt{3}} f(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 140 leaves):

$$-\frac{2}{3} e \operatorname{ArcTanh}[\sqrt{1-x^3}] + \frac{2 f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1-x^3}}$$

Problem 170: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{-1+x^3}} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$\frac{2}{3} e \operatorname{ArcTan}[\sqrt{-1+x^3}] - \frac{2\sqrt{2-\sqrt{3}} f(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 136 leaves):

$$\frac{2}{3} e \operatorname{ArcTan}[\sqrt{-1+x^3}] + \frac{2f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x\right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}}$$

Problem 171: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{x\sqrt{-1-x^3}} dx$$

Optimal (type 4, 131 leaves, 6 steps):

$$\frac{2}{3} e \operatorname{ArcTan}[\sqrt{-1-x^3}] + \frac{2\sqrt{2-\sqrt{3}} f(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 138 leaves):

$$\frac{2}{3} e \operatorname{ArcTan}[\sqrt{-1-x^3}] - \frac{2f \left((-1)^{1/3} - x\right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x\right)}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1-x^3}}$$

Problem 172: Unable to integrate problem.

$$\int \frac{c-dx}{(c+dx)(2c^3+d^3x^3)^{1/3}} dx$$

Optimal (type 3, 95 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2(2c+dx)}{(2c^3+d^3x^3)^{2/3}}}{\sqrt{3}}\right]}{d} - \frac{\operatorname{Log}[c+dx]}{d} + \frac{3 \operatorname{Log}\left[d(2c+dx) - d(2c^3+d^3x^3)^{1/3}\right]}{2d}$$

Result (type 8, 33 leaves):

$$\int \frac{c-dx}{(c+dx)(2c^3+d^3x^3)^{1/3}} dx$$

Problem 173: Unable to integrate problem.

$$\int \frac{e+fx}{(c+dx)(-c^3+d^3x^3)^{1/3}} dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$\frac{f \operatorname{ArcTan}\left[\frac{1+\frac{2dx}{(-c^3+d^3x^3)^{2/3}}}{\sqrt{3}}\right]}{\sqrt{3}d^2} + \frac{\sqrt{3}(de-cf) \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3}(c-dx)}{(-c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3}cd^2} + \frac{(de-cf) \operatorname{Log}\left[\frac{(c-dx)(c+dx)^2}{4 \times 2^{1/3}cd^2}\right]}{4 \times 2^{1/3}cd^2} - \frac{f \operatorname{Log}\left[-dx + (-c^3+d^3x^3)^{1/3}\right]}{2d^2} - \frac{3(de-cf) \operatorname{Log}\left[d(c-dx) + 2^{2/3}d(-c^3+d^3x^3)^{1/3}\right]}{4 \times 2^{1/3}cd^2}$$

Result (type 8, 32 leaves):

$$\int \frac{e+fx}{(c+dx)(-c^3+d^3x^3)^{1/3}} dx$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} dx$$

Optimal (type 5, 209 leaves, 3 steps):

$$\frac{a^2 d (2b^3 c - a^3 d) (a+bx)^{1+n}}{b^6 (1+n)} - \frac{a d (4b^3 c - 5a^3 d) (a+bx)^{2+n}}{b^6 (2+n)} + \frac{2 d (b^3 c - 5a^3 d) (a+bx)^{3+n}}{b^6 (3+n)} + \frac{10 a^2 d^2 (a+bx)^{4+n}}{b^6 (4+n)} - \frac{5 a d^2 (a+bx)^{5+n}}{b^6 (5+n)} + \frac{d^2 (a+bx)^{6+n}}{b^6 (6+n)} - \frac{c^2 (a+bx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1+\frac{bx}{a}\right]}{a (1+n)}$$

Result (type 5, 420 leaves):

$$\begin{aligned}
& (a + b x)^n \left(\frac{1}{b^3 (1+n) (2+n) (3+n)} \right. \\
& 2 c d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + b^3 (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \frac{1}{b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)} d^2 \left(1 + \frac{b x}{a} \right)^{-n} \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& 20 a^3 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - 5 a^2 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + a b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \\
& \left. b^6 (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \frac{c^2 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{a}{b x} \right]}{n} \Big)
\end{aligned}$$

Problem 182: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 459 leaves, 2 steps):

$$\begin{aligned}
& \frac{a^2 (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{12} (1+n)} - \frac{a (2 b^3 c - 11 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{12} (2+n)} + \frac{(b^3 c - a^3 d) (b^6 c^2 - 29 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{3+n}}{b^{12} (3+n)} + \\
& \frac{3 a^2 d (10 b^6 c^2 - 56 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{4+n}}{b^{12} (4+n)} - \frac{15 a d (b^6 c^2 - 14 a^3 b^3 c d + 22 a^6 d^2) (a + b x)^{5+n}}{b^{12} (5+n)} + \\
& \frac{3 d (b^6 c^2 - 56 a^3 b^3 c d + 154 a^6 d^2) (a + b x)^{6+n}}{b^{12} (6+n)} + \frac{42 a^2 d^2 (2 b^3 c - 11 a^3 d) (a + b x)^{7+n}}{b^{12} (7+n)} - \frac{6 a d^2 (4 b^3 c - 55 a^3 d) (a + b x)^{8+n}}{b^{12} (8+n)} + \\
& \frac{3 d^2 (b^3 c - 55 a^3 d) (a + b x)^{9+n}}{b^{12} (9+n)} + \frac{55 a^2 d^3 (a + b x)^{10+n}}{b^{12} (10+n)} - \frac{11 a d^3 (a + b x)^{11+n}}{b^{12} (11+n)} + \frac{d^3 (a + b x)^{12+n}}{b^{12} (12+n)}
\end{aligned}$$

Result (type 3, 1134 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} \left(-39916800 a^{11} d^3 + 39916800 a^{10} b d^3 (1+n) x - \right. \right. \\
& 19958400 a^9 b^2 d^3 (2+3n+n^2) x^2 + 120960 a^8 b^3 d^2 \left(c (1320+362n+33n^2+n^3) + 55d (6+11n+6n^2+n^3) x^3 \right) - \\
& 30240 a^7 b^4 d^2 (1+n) x \left(4c (1320+362n+33n^2+n^3) + 55d (24+26n+9n^2+n^3) x^3 \right) + \\
& 30240 a^6 b^5 d^2 (2+3n+n^2) x^2 \left(2c (1320+362n+33n^2+n^3) + 11d (60+47n+12n^2+n^3) x^3 \right) - \\
& 360 a^5 b^6 d \left(c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \right. \\
& 56cd (7920+16692n+12100n^2+3861n^3+571n^4+39n^5+n^6) x^3 + 154d^2 (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^6 \left. \right) + \\
& 360 a^4 b^7 d (1+n) x \left(c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \right. \\
& 14cd (31680+43008n+22084n^2+5460n^3+685n^4+42n^5+n^6) x^3 + 22d^2 (5040+8028n+5104n^2+1665n^3+295n^4+27n^5+n^6) x^6 \left. \right) - \\
& 18 a^3 b^8 d (2+3n+n^2) x^2 \left(10c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \right. \\
& 56cd (79200+83760n+34834n^2+7275n^3+805n^4+45n^5+n^6) x^3 + 55d^2 (20160+24552n+12154n^2+3135n^3+445n^4+33n^5+n^6) x^6 \left. \right) + \\
& b^{11} \left(246400+593520n+541508n^2+251352n^3+66489n^4+10440n^5+962n^6+48n^7+n^8 \right) x^2 \left(c^3 (648+234n+27n^2+n^3) + \right. \\
& 3c^2d (324+171n+24n^2+n^3) x^3 + 3cd^2 (216+126n+21n^2+n^3) x^6 + d^3 (162+99n+18n^2+n^3) x^9 \left. \right) - a b^{10} (280+418n+159n^2+22n^3+n^4) x \\
& \left(2c^3 (285120+221544n+70254n^2+11645n^3+1065n^4+51n^5+n^6) + 15c^2d (57024+70920n+32574n^2+7115n^3+801n^4+45n^5+n^6) x^3 + \right. \\
& 24cd^2 (23760+32652n+17160n^2+4421n^3+591n^4+39n^5+n^6) x^6 + 11d^3 (12960+18612n+10404n^2+2915n^3+435n^4+33n^5+n^6) x^9 \left. \right) + \\
& 2a^2 b^9 \left(c^3 (79833600+101378880n+56231712n^2+17893196n^3+3602088n^4+476049n^5+41328n^6+2274n^7+72n^8+n^9) + \right. \\
& 30c^2d (3991680+9925488n+9476652n^2+4665572n^3+1332327n^4+233481n^5+25518n^6+1698n^7+63n^8+n^9) x^3 + \\
& 84cd^2 (950400+2589120n+2806008n^2+1617020n^3+552426n^4+116949n^5+15432n^6+1230n^7+54n^8+n^9) x^6 + \\
& \left. \left. 55d^3 (362880+1026576n+1172700n^2+723680n^3+269325n^4+63273n^5+9450n^6+870n^7+45n^8+n^9) x^9 \right) \right) \Big/ \\
& (b^{12} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) (10+n) \\
& (11+n) \\
& (12+n))
\end{aligned}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 396 leaves, 2 steps):

$$\begin{aligned}
& - \frac{a (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{11} (1+n)} + \frac{(b^3 c - 10 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{11} (2+n)} + \frac{9 a^2 d (2 b^3 c - 5 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{11} (3+n)} - \\
& \frac{3 a d (4 b^6 c^2 - 35 a^3 b^3 c d + 40 a^6 d^2) (a + b x)^{4+n}}{b^{11} (4+n)} + \frac{3 d (b^6 c^2 - 35 a^3 b^3 c d + 70 a^6 d^2) (a + b x)^{5+n}}{b^{11} (5+n)} + \frac{63 a^2 d^2 (b^3 c - 4 a^3 d) (a + b x)^{6+n}}{b^{11} (6+n)} - \\
& \frac{21 a d^2 (b^3 c - 10 a^3 d) (a + b x)^{7+n}}{b^{11} (7+n)} + \frac{3 d^2 (b^3 c - 40 a^3 d) (a + b x)^{8+n}}{b^{11} (8+n)} + \frac{45 a^2 d^3 (a + b x)^{9+n}}{b^{11} (9+n)} - \frac{10 a d^3 (a + b x)^{10+n}}{b^{11} (10+n)} + \frac{d^3 (a + b x)^{11+n}}{b^{11} (11+n)}
\end{aligned}$$

Result (type 3, 903 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} (3\,628\,800 a^{10} d^3 - 3\,628\,800 a^9 b d^3 (1+n) x + \right. \\
& \quad 1\,814\,400 a^8 b^2 d^3 (2+3n+n^2) x^2 - 15\,120 a^7 b^3 d^2 (c (990+299n+30n^2+n^3) + 40d (6+11n+6n^2+n^3) x^3) + \\
& \quad 15\,120 a^6 b^4 d^2 (1+n) x (c (990+299n+30n^2+n^3) + 10d (24+26n+9n^2+n^3) x^3) - 7560 a^5 b^5 d^2 (2+3n+n^2) x^2 \\
& \quad (c (990+299n+30n^2+n^3) + 4d (60+47n+12n^2+n^3) x^3) + 72 a^4 b^6 d (c^2 (332\,640+245\,004n+74\,524n^2+11\,985n^3+10\,75n^4+51n^5+n^6) + \\
& \quad 35 c d (5940+12\,684n+9409n^2+3120n^3+490n^4+36n^5+n^6) x^3 + 70 d^2 (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^6) - \\
& \quad 18 a^3 b^7 d (1+n) x (4 c^2 (332\,640+245\,004n+74\,524n^2+11\,985n^3+10\,75n^4+51n^5+n^6) + \\
& \quad 35 c d (23\,760+32\,916n+17\,404n^2+4485n^3+595n^4+39n^5+n^6) x^3 + 40 d^2 (5040+8028n+5104n^2+1665n^3+295n^4+27n^5+n^6) x^6) + \\
& \quad 18 a^2 b^8 d (2+3n+n^2) x^2 (2 c^2 (332\,640+245\,004n+74\,524n^2+11\,985n^3+10\,75n^4+51n^5+n^6) + \\
& \quad 7 c d (59\,400+64\,470n+27\,733n^2+6048n^3+706n^4+42n^5+n^6) x^3 + 5 d^2 (20\,160+24\,552n+12\,154n^2+3135n^3+445n^4+33n^5+n^6) x^6) + \\
& \quad b^{10} (45\,360+95\,436n+72\,180n^2+27\,109n^3+5620n^4+654n^5+40n^6+n^7) x (c^3 (440+183n+24n^2+n^3) + \\
& \quad 3 c^2 d (176+126n+21n^2+n^3) x^3 + 3 c d^2 (110+87n+18n^2+n^3) x^6 + d^3 (80+66n+15n^2+n^3) x^9) - a b^9 (162+99n+18n^2+n^3) \\
& \quad (c^3 (123\,200+111\,960n+41\,214n^2+7875n^3+825n^4+45n^5+n^6) + 12 c^2 d (12\,320+24\,132n+15\,600n^2+4341n^3+591n^4+39n^5+n^6) x^3 + \\
& \quad 21 c d^2 (4400+9420n+7068n^2+2427n^3+411n^4+33n^5+n^6) x^6 + 10 d^3 (2240+4968n+3954n^2+1485n^3+285n^4+27n^5+n^6) x^9) \Big) / \\
& (b^{11} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) (10+n) (11+n))
\end{aligned}$$

Problem 184: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 337 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{10} (1+n)} + \frac{9 a^2 d (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{10} (2+n)} - \frac{9 a d (b^3 c - 4 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{10} (3+n)} + \\
& \frac{3 d (b^6 c^2 - 20 a^3 b^3 c d + 28 a^6 d^2) (a + b x)^{4+n}}{b^{10} (4+n)} + \frac{9 a^2 d^2 (5 b^3 c - 14 a^3 d) (a + b x)^{5+n}}{b^{10} (5+n)} - \frac{18 a d^2 (b^3 c - 7 a^3 d) (a + b x)^{6+n}}{b^{10} (6+n)} + \\
& \frac{3 d^2 (b^3 c - 28 a^3 d) (a + b x)^{7+n}}{b^{10} (7+n)} + \frac{36 a^2 d^3 (a + b x)^{8+n}}{b^{10} (8+n)} - \frac{9 a d^3 (a + b x)^{9+n}}{b^{10} (9+n)} + \frac{d^3 (a + b x)^{10+n}}{b^{10} (10+n)}
\end{aligned}$$

Result (type 3, 706 leaves):

$$\left((a + bx)^{1+n} \right. \\
\left. \begin{aligned} & (-362880 a^9 d^3 + 362880 a^8 b d^3 (1+n) x - 181440 a^7 b^2 d^3 (2+3n+n^2) x^2 + 2160 a^6 b^3 d^2 (c(720+242n+27n^2+n^3) + 28d(6+11n+6n^2+n^3) x^3) - \\ & 2160 a^5 b^4 d^2 (1+n) x (c(720+242n+27n^2+n^3) + 7d(24+26n+9n^2+n^3) x^3) + \\ & 216 a^4 b^5 d^2 (2+3n+n^2) x^2 (5c(720+242n+27n^2+n^3) + 14d(60+47n+12n^2+n^3) x^3) - 9 a b^8 d (80+146n+81n^2+16n^3+n^4) \\ & x^2 (c^2(3780+1968n+379n^2+32n^3+n^4) + 2cd(1080+858n+235n^2+26n^3+n^4) x^3 + d^2(504+450n+145n^2+20n^3+n^4) x^6) - \\ & 18 a^3 b^6 d (c^2(151200+127860n+44524n^2+8175n^3+835n^4+45n^5+n^6) + 20cd(4320+9372n+7144n^2+2475n^3+415n^4+33n^5+n^6) x^3 + \\ & 28d^2(720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^6) + 18 a^2 b^7 d (1+n) x \\ & (c^2(151200+127860n+44524n^2+8175n^3+835n^4+45n^5+n^6) + 5cd(17280+24528n+13420n^2+3624n^3+511n^4+36n^5+n^6) x^3 + \\ & 4d^2(5040+8028n+5104n^2+1665n^3+295n^4+27n^5+n^6) x^6) + b^9(12960+18612n+10404n^2+2915n^3+435n^4+33n^5+n^6) \\ & (c^3(280+138n+21n^2+n^3) + 3c^2d(70+87n+18n^2+n^3) x^3 + 3cd^2(40+54n+15n^2+n^3) x^6 + d^3(28+39n+12n^2+n^3) x^9) \Big) \Big/ \\ & (b^{10}(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)(8+n)(9+n)(10+n)) \end{aligned} \right)$$

Problem 185: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^n (c + dx^3)^3}{x} dx$$

Optimal (type 5, 358 leaves, 3 steps):

$$\frac{a^2 d (3 b^6 c^2 - 3 a^3 b^3 c d + a^6 d^2) (a + bx)^{1+n}}{b^9 (1+n)} - \frac{a d (6 b^6 c^2 - 15 a^3 b^3 c d + 8 a^6 d^2) (a + bx)^{2+n}}{b^9 (2+n)} + \frac{d (3 b^6 c^2 - 30 a^3 b^3 c d + 28 a^6 d^2) (a + bx)^{3+n}}{b^9 (3+n)} + \\
\frac{2 a^2 d^2 (15 b^3 c - 28 a^3 d) (a + bx)^{4+n}}{b^9 (4+n)} - \frac{5 a d^2 (3 b^3 c - 14 a^3 d) (a + bx)^{5+n}}{b^9 (5+n)} + \frac{d^2 (3 b^3 c - 56 a^3 d) (a + bx)^{6+n}}{b^9 (6+n)} + \\
\frac{28 a^2 d^3 (a + bx)^{7+n}}{b^9 (7+n)} - \frac{8 a d^3 (a + bx)^{8+n}}{b^9 (8+n)} + \frac{d^3 (a + bx)^{9+n}}{b^9 (9+n)} - \frac{c^3 (a + bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{bx}{a}\right]}{a (1+n)}$$

Result (type 5, 856 leaves):

$$\begin{aligned}
& (a + b x)^n \left(\frac{1}{b^3 (1+n) (2+n) (3+n)} \right. \\
& 3 c^2 d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + b^3 (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \frac{1}{b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)} 3 c d^2 \left(1 + \frac{b x}{a} \right)^{-n} \\
& \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + 20 a^3 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - 5 a^2 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. a b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + b^6 (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \frac{1}{b^9 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n)} d^3 \left(1 + \frac{b x}{a} \right)^{-n} \\
& \left(-40320 a^8 b n x \left(1 + \frac{b x}{a} \right)^n + 20160 a^7 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n - 6720 a^6 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& 1680 a^5 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n - 336 a^4 b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \\
& 56 a^3 b^6 n (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 8 a^2 b^7 n (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^7 \left(1 + \frac{b x}{a} \right)^n + \\
& \left. a b^8 n (5040+13068n+13132n^2+6769n^3+1960n^4+322n^5+28n^6+n^7) x^8 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. b^9 (40320+109584n+118124n^2+67284n^3+22449n^4+4536n^5+546n^6+36n^7+n^8) x^9 \left(1 + \frac{b x}{a} \right)^n + 40320 a^9 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \left. \frac{c^3 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{a}{b x} \right]}{n} \right)
\end{aligned}$$

Problem 186: Result is not expressed in closed-form.

$$\int \frac{x^5 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 324 leaves, 7 steps):

$$\begin{aligned}
& \frac{e^2 (e + f x)^{1+n}}{b f^3 (1+n)} - \frac{2 e (e + f x)^{2+n}}{b f^3 (2+n)} + \frac{(e + f x)^{3+n}}{b f^3 (3+n)} + \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f} \right]}{3 b^{5/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f} \right]}{3 b^{5/3} (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} + \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f} \right]}{3 b^{5/3} (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)}
\end{aligned}$$

Result (type 7, 423 leaves):

$$\frac{1}{3 b f^3} (e + f x)^n \left(\frac{3 \left(-2 e^2 f n x + e f^2 n (1+n) x^2 + f^3 (2+3n+n^2) x^3 + e^3 \left(2 - 2 \left(1 + \frac{f x}{e} \right)^{-n} \right) \right)}{6 + 11 n + 6 n^2 + n^3} - \right.$$

$$\frac{1}{b n} a f^3 \left(e^2 \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1} \right] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right] - \right.$$

$$2 e \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1} \right] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \& \right] +$$

$$\left. \left. \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1} \right] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \& \right] \right) \right)$$

Problem 187: Result is not expressed in closed-form.

$$\int \frac{x^4 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 332 leaves, 7 steps):

$$-\frac{e (e + f x)^{1+n}}{b f^2 (1+n)} + \frac{(e + f x)^{2+n}}{b f^2 (2+n)} - \frac{a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f} \right]}{3 b^{4/3} (b^{1/3} e - a^{1/3} f) (1+n)} +$$

$$\frac{(-1)^{1/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f} \right]}{3 b^{4/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n)} +$$

$$\frac{(-1)^{2/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f} \right]}{3 b^{4/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n)}$$

Result (type 7, 298 leaves):

$$\frac{1}{3 b f^2} (e + f x)^n \left(-\frac{3 \left(-e f n x - f^2 (1+n) x^2 + e^2 \left(1 - \left(1 + \frac{f x}{e} \right)^{-n} \right) \right)}{2 + 3 n + n^2} + \right.$$

$$\frac{a e f^3 \operatorname{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-1} \right] \left(\frac{e+f x}{e+f x-1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right]}{b n} -$$

$$\left. \frac{a f^3 \operatorname{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-1} \right] \left(\frac{e+f x}{e+f x-1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \& \right]}{b n} \right)$$

Problem 188: Result is not expressed in closed-form.

$$\int \frac{x^3 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 293 leaves, 7 steps):

$$\frac{(e + f x)^{1+n}}{b f (1+n)} + \frac{a^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f} \right]}{3 b (b^{1/3} e - a^{1/3} f) (1+n)} +$$

$$\frac{a^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f} \right]}{3 b \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n)} - \frac{a^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f} \right]}{3 b \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n)}$$

Result (type 7, 142 leaves):

$$\frac{(e + f x)^n \left(\frac{3 b (e+f x)}{1+n} - \frac{a f^3 \operatorname{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\#1}{e+f x-1} \right] \left(\frac{e+f x}{e+f x-1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right]}{n} \right)}{3 b^2 f}$$

Problem 189: Result is not expressed in closed-form.

$$\int \frac{x^2 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3}(e+fx)}{b^{1/3}e-a^{1/3}f}\right]}{3b^{2/3}(b^{1/3}e-a^{1/3}f)(1+n)} \\
& - \frac{(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3}(e+fx)}{b^{1/3}e+(-1)^{1/3}a^{1/3}f}\right]}{3b^{2/3}(b^{1/3}e+(-1)^{1/3}a^{1/3}f)(1+n)} - \frac{(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3}(e+fx)}{b^{1/3}e-(-1)^{2/3}a^{1/3}f}\right]}{3b^{2/3}(b^{1/3}e-(-1)^{2/3}a^{1/3}f)(1+n)}
\end{aligned}$$

Result (type 7, 337 leaves):

$$\begin{aligned}
& \frac{1}{3bn} (e+fx)^n \left(e^2 \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right] - \right. \\
& 2 e \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&\right] + \\
& \left. \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \&\right] \right)
\end{aligned}$$

Problem 190: Result is not expressed in closed-form.

$$\int \frac{x (e+fx)^n}{a+bx^3} dx$$

Optimal (type 5, 288 leaves, 5 steps):

$$\begin{aligned}
& \frac{(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3}(e+fx)}{b^{1/3}e-a^{1/3}f}\right]}{3a^{1/3}b^{1/3}(b^{1/3}e-a^{1/3}f)(1+n)} \\
& - \frac{(-1)^{1/3}(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{2/3}b^{1/3}(e+fx)}{(-1)^{2/3}b^{1/3}e-a^{1/3}f}\right]}{3a^{1/3}b^{1/3}\left((-1)^{2/3}b^{1/3}e-a^{1/3}f\right)(1+n)} - \frac{(-1)^{2/3}(e+fx)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{1/3}b^{1/3}(e+fx)}{(-1)^{1/3}b^{1/3}e+a^{1/3}f}\right]}{3a^{1/3}b^{1/3}\left((-1)^{1/3}b^{1/3}e+a^{1/3}f\right)(1+n)}
\end{aligned}$$

Result (type 7, 229 leaves):

$$\begin{aligned}
& - \frac{1}{3bn} f (e+fx)^n \left(e \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right] - \right. \\
& \left. \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&\right] \right)
\end{aligned}$$

Problem 191: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 263 leaves, 5 steps):

$$\frac{(e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} (b^{1/3} e - a^{1/3} f) (1 + n)} - \frac{(e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{(-1)^{2/3} b^{1/3} (e + f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f\right) (1 + n)} + \frac{(e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{(-1)^{1/3} b^{1/3} (e + f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right]}{3 a^{2/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f\right) (1 + n)}$$

Result (type 7, 122 leaves):

$$\frac{1}{3 b n} f^2 (e + f x)^n \operatorname{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1}\right] \left(\frac{e + f x}{e + f x - \#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right]$$

Problem 192: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x (a + b x^3)} dx$$

Optimal (type 5, 300 leaves, 8 steps):

$$\frac{b^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - a^{1/3} f}\right]}{3 a (b^{1/3} e - a^{1/3} f) (1 + n)} + \frac{b^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}\right]}{3 a (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1 + n)} + \frac{b^{1/3} (e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}\right]}{3 a (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1 + n)} - \frac{(e + f x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, 1 + \frac{f x}{e}\right]}{a e (1 + n)}$$

Result (type 7, 377 leaves):

$$\frac{1}{3 a n} (e + f x)^n \left(3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{e}{f x} \right] - \right.$$

$$e^2 \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1} \right] \left(\frac{e + f x}{e + f x - \#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right] +$$

$$2 e \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1} \right] \left(\frac{e + f x}{e + f x - \#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \& \right] -$$

$$\left. \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1} \right] \left(\frac{e + f x}{e + f x - \#1} \right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \& \right] \right)$$

Problem 193: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x^2 (a + b x^3)} dx$$

Optimal (type 5, 326 leaves, 8 steps):

$$-\frac{b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - a^{1/3} f} \right]}{3 a^{4/3} (b^{1/3} e - a^{1/3} f) (1 + n)} + \frac{(-1)^{1/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1 + n, 2 + n, \frac{(-1)^{2/3} b^{1/3} (e + f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f} \right]}{3 a^{4/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1 + n)} +$$

$$\frac{(-1)^{2/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1 + n, 2 + n, \frac{(-1)^{1/3} b^{1/3} (e + f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f} \right]}{3 a^{4/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1 + n)} + \frac{f (e + f x)^{1+n} \text{Hypergeometric2F1} \left[2, 1 + n, 2 + n, 1 + \frac{f x}{e} \right]}{a e^2 (1 + n)}$$

Result (type 7, 280 leaves):

$$\frac{1}{3 a} (e + f x)^n \left(\frac{3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1} \left[1 - n, -n, 2 - n, -\frac{e}{f x} \right]}{(-1 + n) x} + \right.$$

$$\frac{e f \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1} \right] \left(\frac{e + f x}{e + f x - \#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right]}{n} -$$

$$\left. \frac{f \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1} \right] \left(\frac{e + f x}{e + f x - \#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \& \right]}{n} \right)$$

Problem 194: Result is not expressed in closed-form.

$$\int \frac{x^2 (c + d x)^{1+n}}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps):

$$\frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/3} (c + d x)}{b^{1/3} c - a^{1/3} d}\right]}{3 b^{2/3} (b^{1/3} c - a^{1/3} d) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/3} (c + d x)}{b^{1/3} c + (-1)^{1/3} a^{1/3} d}\right]}{3 b^{2/3} (b^{1/3} c + (-1)^{1/3} a^{1/3} d) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/3} (c + d x)}{b^{1/3} c - (-1)^{2/3} a^{1/3} d}\right]}{3 b^{2/3} (b^{1/3} c - (-1)^{2/3} a^{1/3} d) (2 + n)}$$

Result (type 7, 375 leaves):

$$\frac{1}{3 b^2 n (1 + n)} (c + d x)^n \left((b c^3 - a d^3) (1 + n) \operatorname{RootSum}\left[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n}}{c^2 - 2 c \#1 + \#1^2} \&\right] + b \left(3 n (c + d x) - 2 c^2 (1 + n) \operatorname{RootSum}\left[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1}{c^2 - 2 c \#1 + \#1^2} \&\right] + c (1 + n) \operatorname{RootSum}\left[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1^2}{c^2 - 2 c \#1 + \#1^2} \&\right] \right)$$

Problem 195: Unable to integrate problem.

$$\int \frac{x^m (e + f x)^n}{a + b x^3} dx$$

Optimal (type 6, 211 leaves, 8 steps):

$$\frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \operatorname{AppellF1}\left[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{b^{1/3} x}{a^{1/3}}\right]}{3 a (1 + m)} + \frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \operatorname{AppellF1}\left[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, \frac{(-1)^{1/3} b^{1/3} x}{a^{1/3}}\right]}{3 a (1 + m)} + \frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \operatorname{AppellF1}\left[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{(-1)^{2/3} b^{1/3} x}{a^{1/3}}\right]}{3 a (1 + m)}$$

Result (type 8, 22 leaves):

$$\int \frac{x^m (e + f x)^n}{a + b x^3} dx$$

Problem 196: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c + d x^3}}{a + b x} dx$$

Optimal (type 4, 1482 leaves, 13 steps):

$$\begin{aligned} & \frac{2\sqrt{c+dx^3}}{3b} - \frac{2ad^{1/3}\sqrt{c+dx^3}}{b^2\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)} - \left(c^{1/6}\sqrt{bc^{1/3}-ad^{1/3}}\sqrt{b^2c^{2/3}+ab^{1/3}d^{1/3}+a^2d^{2/3}}(c^{1/3}+d^{1/3}x) \sqrt{\frac{c^{2/3}\left(1-\frac{d^{1/3}x}{c^{1/3}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}} \right. \\ & \left. \operatorname{ArcTanh}\left[\frac{\sqrt{2-\sqrt{3}}\sqrt{b^2c^{2/3}+ab^{1/3}d^{1/3}+a^2d^{2/3}}\sqrt{1-\frac{\left((1-\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}}}{3^{1/4}\sqrt{b}c^{1/6}\sqrt{bc^{1/3}-ad^{1/3}}\sqrt{7-4\sqrt{3}+\frac{\left((1-\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}}}\right] \Big/ \left(b^{5/2}\sqrt{\frac{c^{1/3}(c^{1/3}+d^{1/3}x)}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3} \right) + \right. \\ & \left. \left(3^{1/4}\sqrt{2-\sqrt{3}}a^{1/3}d^{1/3}(c^{1/3}+d^{1/3}x)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})c^{1/3}+d^{1/3}x}{(1+\sqrt{3})c^{1/3}+d^{1/3}x}\right], -7-4\sqrt{3}\right] \right) \Big/ \right. \\ & \left. \left(b^2\sqrt{\frac{c^{1/3}(c^{1/3}+d^{1/3}x)}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3} \right) + \right. \\ & \left. \left(2\sqrt{2+\sqrt{3}}a\left((1-\sqrt{3})b^{1/3}+ad^{1/3}\right)d^{1/3}(c^{1/3}+d^{1/3}x)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})c^{1/3}+d^{1/3}x}{(1+\sqrt{3})c^{1/3}+d^{1/3}x}\right], -7-4\sqrt{3}\right] \right) \Big/ \right. \\ & \left. \left(3^{1/4}b^3\sqrt{\frac{c^{1/3}(c^{1/3}+d^{1/3}x)}{\left((1+\sqrt{3})c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3} \right) - \right. \end{aligned}$$

$$\begin{aligned}
& \left(2 \sqrt{2 + \sqrt{3}} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{((1 + \sqrt{3}) c^{1/3} + d^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x}\right], -7 - 4\sqrt{3}\right] \right) / \\
& \left(3^{1/4} b^3 \left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3} \right) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{((1 + \sqrt{3}) c^{1/3} + d^{1/3} x)^2}} \sqrt{c + d x^3} \right) - \\
& \left(4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} c^{1/3} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} \left(1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}\right)}{((1 + \sqrt{3}) c^{1/3} + d^{1/3} x)^2}} \operatorname{EllipticPi}\left[\frac{\left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3}\right)^2}{\left((1 - \sqrt{3}) b c^{1/3} - a d^{1/3}\right)^2}, \right. \right. \\
& \left. \left. -\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x}\right], -7 - 4\sqrt{3}\right] \right) / \left(b^2 (2 b^2 c^{2/3} + 2 a b c^{1/3} d^{1/3} - a^2 d^{2/3}) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{((1 + \sqrt{3}) c^{1/3} + d^{1/3} x)^2}} \sqrt{c + d x^3} \right)
\end{aligned}$$

Result (type 4, 820 leaves):

$$\begin{aligned}
& \frac{1}{3 b \sqrt{c+d x^3}} 2 \left(c+d x^3 - \frac{1}{b^2 \sqrt{\frac{c^{1/3}+(-1)^{2/3} d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}}} \right. \\
& 3^{3/4} a^2 d^{2/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{\frac{c^{1/3}+d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}} \sqrt{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c^{1/3}+(-1)^{2/3} d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}\right] + \\
& \frac{1}{b \sqrt{\frac{c^{1/3}+(-1)^{2/3} d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}}} 3^{3/4} a c^{1/3} d^{1/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2 i d^{1/3} x}{c^{1/3}}} \sqrt{\frac{i \left(1 + \frac{d^{1/3} x}{c^{1/3}}\right)}{3 i + \sqrt{3}}} \\
& \left(\left(-1 + (-1)^{2/3} \right) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] + \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] \right) - \\
& \frac{1}{(-1)^{1/3} b c^{1/3} + a d^{1/3}} 3 i b c^{4/3} \sqrt{\frac{c^{1/3}+d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}} \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \\
& \text{EllipticPi}\left[\frac{i \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \text{ArcSin}\left[\sqrt{\frac{c^{1/3}+(-1)^{2/3} d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}\right] + \left((-1)^{1/3} \sqrt{3} (1+(-1)^{1/3}) a^3 c^{1/3} d \sqrt{\frac{c^{1/3}+d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}} \right. \\
& \left. \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \text{EllipticPi}\left[\frac{i \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \text{ArcSin}\left[\sqrt{\frac{c^{1/3}+(-1)^{2/3} d^{1/3} x}{(1+(-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(b^2 \left((-1)^{1/3} b c^{1/3} + a d^{1/3} \right) \right)
\end{aligned}$$

Problem 197: Unable to integrate problem.

$$\int \frac{(d^3 + e^3 x^3)^p}{d + e x} dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{(d^3 + e^3 x^3)^p \left(1 + \frac{2(d+ex)}{(-3+i\sqrt{3})d}\right)^{-p} \left(1 - \frac{2(d+ex)}{(3+i\sqrt{3})d}\right)^{-p} \text{AppellF1}\left[p, -p, -p, 1+p, -\frac{2(d+ex)}{(-3+i\sqrt{3})d}, \frac{2(d+ex)}{(3+i\sqrt{3})d}\right]}{e^p}$$

Result (type 8, 23 leaves):

$$\int \frac{(d^3 + e^3 x^3)^p}{d + e x} dx$$

Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 - 2x - x^2}{(2 + x^2) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2 \text{ArcTan}\left[\frac{1+x}{\sqrt{1+x^3}}\right]$$

Result (type 4, 296 leaves):

$$\frac{1}{3\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \left(\frac{\sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}-x) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3}x} - \right.$$

$$\left. \frac{3i(-i+\sqrt{2}) \text{EllipticPi}\left[\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}+\sqrt{2}} + \right.$$

$$\left. \frac{3(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}) \text{EllipticPi}\left[\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}} \right)$$

Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 + 2x - x^2}{(2 + x^2) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-2 \operatorname{ArcTan} \left[\frac{1 - x}{\sqrt{1 - x^3}} \right]$$

Result (type 4, 280 leaves):

$$\frac{1}{3 \sqrt{1 - x^3}} 2 \sqrt{\frac{1 - x}{1 + (-1)^{1/3}}} \sqrt{1 + x + x^2} \left(\frac{\sqrt{3} (1 + (-1)^{1/3}) ((-1)^{1/3} + x) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right]}{-1 + (-1)^{2/3} x} + \right.$$

$$\left. \frac{6 (1 + i \sqrt{2}) \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{-i - 2 \sqrt{2} + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right]}{i + 2 \sqrt{2} - \sqrt{3}} + \right.$$

$$\left. \frac{3 (1 - i \sqrt{2}) \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{-i + 2 \sqrt{2} + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right]}{(-1)^{5/6} - \sqrt{2}} \right)$$

Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 + 2x - x^2}{(2 + x^2) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-2 \operatorname{ArcTanh} \left[\frac{1 - x}{\sqrt{-1 + x^3}} \right]$$

Result (type 4, 278 leaves):

$$\frac{1}{3\sqrt{-1+x^3}} 2\sqrt{\frac{1-x}{1+(-1)^{1/3}}}\sqrt{1+x+x^2} \left(\frac{\sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}+x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3}x} + \right.$$

$$\frac{6(1+i\sqrt{2}) \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{i+2\sqrt{2}-\sqrt{3}} +$$

$$\left. \frac{3(1-i\sqrt{2}) \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}-\sqrt{2}} \right)$$

Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$2 \operatorname{ArcTanh}\left[\frac{1+x}{\sqrt{-1-x^3}}\right]$$

Result (type 4, 298 leaves):

$$\frac{1}{3\sqrt{-1-x^3}} 2\sqrt{\frac{1+x}{1+(-1)^{1/3}}}\sqrt{1-x+x^2} \left(\frac{\sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}-x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3}x} - \right.$$

$$\frac{3i(-i+\sqrt{2}) \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}+\sqrt{2}} +$$

$$\left. \frac{3(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}) \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}} \right)$$

Problem 202: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2x-x^2}{(2+dx+x^2)\sqrt{1+x^3}} dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 424 leaves):

$$\frac{1}{3\sqrt{1+x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2}$$

$$\left(\frac{2\sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}-x) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3}x} - \frac{1}{(2+(-1)^{2/3}+d+(-1)^{1/3}d)\sqrt{-8-4d+d^2}} \right.$$

$$3i \left(\left(8+8(-1)^{1/3} - (1+(-1)^{1/3})d^2 + 4\sqrt{-8-4d+d^2} - 2(-1)^{1/3}\sqrt{-8-4d+d^2} + (1+(-1)^{1/3})d(4+\sqrt{-8-4d+d^2}) \right) \right.$$

$$\left. \text{EllipticPi}\left[\frac{2i\sqrt{3}}{2(-1)^{1/3}+d-\sqrt{-8-4d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left((1+(-1)^{1/3})d^2 + (1+(-1)^{1/3})d(-4+\sqrt{-8-4d+d^2}) - 2(4+4(-1)^{1/3}-2\sqrt{-8-4d+d^2}+(-1)^{1/3}\sqrt{-8-4d+d^2}) \right) \right.$$

$$\left. \left. \text{EllipticPi}\left[\frac{2i\sqrt{3}}{2(-1)^{1/3}+d+\sqrt{-8-4d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{1-d}}$$

Result (type 4, 427 leaves):

$$\frac{1}{3\sqrt{1-x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2}$$

$$\left(\frac{2\sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}+x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3}x} + \frac{1}{(-2-(-1)^{2/3}+d+(-1)^{1/3}d) \sqrt{-8+4d+d^2}} \right.$$

$$3i \left((8+8(-1)^{1/3} - (1+(-1)^{1/3})d^2 - 4\sqrt{-8+4d+d^2} + 2(-1)^{1/3}\sqrt{-8+4d+d^2} + (1+(-1)^{1/3})d(-4+\sqrt{-8+4d+d^2})) \right.$$

$$\operatorname{EllipticPi}\left[\frac{2i\sqrt{3}}{2(-1)^{1/3}-d+\sqrt{-8+4d+d^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] +$$

$$\left. (-8-8(-1)^{1/3} + (1+(-1)^{1/3})d^2 - 4\sqrt{-8+4d+d^2} + 2(-1)^{1/3}\sqrt{-8+4d+d^2} + (1+(-1)^{1/3})d(4+\sqrt{-8+4d+d^2})) \right.$$

$$\left. \operatorname{EllipticPi}\left[-\frac{2i\sqrt{3}}{-2(-1)^{1/3}+d+\sqrt{-8+4d+d^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{1-d}}$$

Result (type 4, 425 leaves):

$$\frac{1}{3 \sqrt{-1+x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2}$$

$$\left(\frac{2 \sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}+x) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3}x} + \frac{1}{(-2-(-1)^{2/3}+d+(-1)^{1/3}d) \sqrt{-8+4d+d^2}} \right.$$

$$3 i \left((8+8(-1)^{1/3} - (1+(-1)^{1/3})d^2 - 4\sqrt{-8+4d+d^2} + 2(-1)^{1/3}\sqrt{-8+4d+d^2} + (1+(-1)^{1/3})d(-4+\sqrt{-8+4d+d^2})) \right.$$

$$\text{EllipticPi}\left[\frac{2 i \sqrt{3}}{2(-1)^{1/3}-d+\sqrt{-8+4d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] +$$

$$\left. (-8-8(-1)^{1/3} + (1+(-1)^{1/3})d^2 - 4\sqrt{-8+4d+d^2} + 2(-1)^{1/3}\sqrt{-8+4d+d^2} + (1+(-1)^{1/3})d(4+\sqrt{-8+4d+d^2})) \right.$$

$$\left. \text{EllipticPi}\left[-\frac{2 i \sqrt{3}}{-2(-1)^{1/3}+d+\sqrt{-8+4d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 205: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2x-x^2}{(2+dx+x^2)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 426 leaves):

$$\frac{1}{3 \sqrt{-1-x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2}$$

$$\left(\frac{2 \sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}-x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3}x} - \frac{1}{(2+(-1)^{2/3}+d+(-1)^{1/3}d) \sqrt{-8-4d+d^2}} \right.$$

$$3 \operatorname{Im} \left(\left(8+8(-1)^{1/3} - (1+(-1)^{1/3})d^2 + 4\sqrt{-8-4d+d^2} - 2(-1)^{1/3}\sqrt{-8-4d+d^2} + (1+(-1)^{1/3})d(4+\sqrt{-8-4d+d^2}) \right) \right.$$

$$\operatorname{EllipticPi}\left[\frac{2 \operatorname{Im} \sqrt{3}}{2(-1)^{1/3}+d-\sqrt{-8-4d+d^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] +$$

$$\left((1+(-1)^{1/3})d^2 + (1+(-1)^{1/3})d(-4+\sqrt{-8-4d+d^2}) - 2(4+4(-1)^{1/3}-2\sqrt{-8-4d+d^2}+(-1)^{1/3}\sqrt{-8-4d+d^2}) \right)$$

$$\left. \operatorname{EllipticPi}\left[\frac{2 \operatorname{Im} \sqrt{3}}{2(-1)^{1/3}+d+\sqrt{-8-4d+d^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+ex)^3 \sqrt{a+cx^4} dx$$

Optimal (type 4, 355 leaves, 11 steps):

$$\frac{3}{4} d^2 e x^2 \sqrt{a+c x^4} + \frac{6 a d e^2 x \sqrt{a+c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{15} d x (5 d^2 + 9 e^2 x^2) \sqrt{a+c x^4} + \frac{e^3 (a+c x^4)^{3/2}}{6 c} +$$

$$\frac{3 a d^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{4 \sqrt{c}} - \frac{6 a^{5/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{a+c x^4}} +$$

$$\frac{a^{3/4} d (5 \sqrt{c} d^2 + 9 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 310 leaves):

$$\frac{1}{60 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a+c x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(10 a^2 e^3 + c^2 x^5 (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3) + a c x (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 20 e^3 x^3) + 45 a \sqrt{c} d^2 e \sqrt{a+c x^4} \right. \right.$$

$$\left. \left. \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] \right) + 72 a^{3/2} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \right.$$

$$\left. 8 a \sqrt{c} d (5 i \sqrt{c} d^2 + 9 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^2 \sqrt{a+c x^4} dx$$

Optimal (type 4, 326 leaves, 10 steps):

$$\frac{1}{2} d e x^2 \sqrt{a+c x^4} + \frac{2 a e^2 x \sqrt{a+c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{15} x (5 d^2 + 3 e^2 x^2) \sqrt{a+c x^4} +$$

$$\frac{a d e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{2 \sqrt{c}} - \frac{2 a^{5/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{a+c x^4}} +$$

$$\frac{a^{3/4} (5 \sqrt{c} d^2 + 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 247 leaves):

$$\frac{1}{30 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} \sqrt{a+c x^4}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(\sqrt{c} x (10 d^2 + 15 d e x + 6 e^2 x^2) (a+c x^4) + 15 a d e \sqrt{a+c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] \right) + \right.$$

$$\left. 12 a^{3/2} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - 4 a (5 i \sqrt{c} d^2 + 3 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x) \sqrt{a+c x^4} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$\frac{1}{3} d x \sqrt{a+c x^4} + \frac{1}{4} e x^2 \sqrt{a+c x^4} + \frac{a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{4 \sqrt{c}} + \frac{a^{3/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 132 leaves):

$$\frac{1}{12} \left(x (4 d + 3 e x) \sqrt{a+c x^4} + \frac{3 a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{\sqrt{c}} - \frac{8 i a d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}} \right)$$

Problem 209: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + c x^4} \, dx$$

Optimal (type 4, 105 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{a + c x^4} + \frac{a^{3/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 89 leaves):

$$\frac{x (a + c x^4) - \frac{2 i a \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 \sqrt{a + c x^4}}$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{d + e x} \, dx$$

Optimal (type 4, 730 leaves, 15 steps):

$$\begin{aligned}
& \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c} dx \sqrt{a+cx^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{\sqrt{-cd^4 - ae^4} \operatorname{ArcTan}\left[\frac{\sqrt{-cd^4 - ae^4} x}{de\sqrt{a+cx^4}}\right]}{2e^3} + \frac{\sqrt{c} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+cx^4}}\right]}{2e^3} \\
& \frac{\sqrt{cd^4 + ae^4} \operatorname{ArcTanh}\left[\frac{ae^2 + cd^2 x^2}{\sqrt{cd^4 + ae^4} \sqrt{a+cx^4}}\right]}{2e^3} + \frac{a^{1/4} c^{1/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{e^2 \sqrt{a+cx^4}} \\
& \frac{a^{1/4} c^{1/4} d \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2\right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2e^4 \sqrt{a+cx^4}} + \\
& \frac{c^{1/4} d (cd^4 + ae^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{1/4} e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+cx^4}} \\
& \left(\frac{(\sqrt{c} d^2 - \sqrt{a} e^2) (cd^4 + ae^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4\sqrt{a}\sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4a^{1/4} c^{1/4} d e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+cx^4}} \right) /
\end{aligned}$$

Result (type 4, 451 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} c^{1/4} d e^4 \sqrt{a+cx^4}} \left(-2 \sqrt{a} c^{3/4} d^2 e^2 \sqrt{1 + \frac{cx^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& 2 c^{3/4} d^2 (i\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{1 + \frac{cx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} \\
& \left. \left(-2 (-1)^{1/4} a^{1/4} (cd^4 + ae^4) \sqrt{1 + \frac{cx^4}{a}} \operatorname{EllipticPi}\left[\frac{i\sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + c^{1/4} d e (ae^2 + ce^2 x^4 + \sqrt{cd^4 + ae^4} \sqrt{a+cx^4}) \right. \right. \\
& \left. \left. \left. \operatorname{Log}[-d^2 + e^2 x^2] + \sqrt{c} d^2 \sqrt{a+cx^4} \operatorname{Log}[cx^2 + \sqrt{c} \sqrt{a+cx^4}] - \sqrt{cd^4 + ae^4} \sqrt{a+cx^4} \operatorname{Log}[ae^2 + cd^2 x^2 + \sqrt{cd^4 + ae^4} \sqrt{a+cx^4}] \right) \right) \right)
\end{aligned}$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{(d + e x)^2} dx$$

Optimal (type 4, 1221 leaves, 32 steps):

$$\begin{aligned}
& \frac{2\sqrt{c} x \sqrt{a+c x^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{d \sqrt{a+c x^4}}{e (d^2 - e^2 x^2)} + \frac{x \sqrt{a+c x^4}}{d^2 - e^2 x^2} + \frac{\sqrt{-c d^4 - a e^4} \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3} - \frac{(c d^4 - a e^4) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3 \sqrt{-c d^4 - a e^4}} \\
& \frac{\sqrt{c} d \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{e^3} + \frac{c d^3 \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{e^3 \sqrt{c d^4 + a e^4}} - \frac{2 a^{1/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{e^2 \sqrt{a+c x^4}} + \\
& \frac{3 a^{1/4} c^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2\right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 e^4 \sqrt{a+c x^4}} - \\
& \frac{c^{1/4} (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} e^4 \sqrt{a+c x^4}} + \\
& \frac{c^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} e^4 \sqrt{a+c x^4}} - \\
& \frac{c^{1/4} (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+c x^4}} + \\
& \frac{(\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d^2 e^4 \sqrt{a+c x^4}} + \\
& \left(\frac{(\sqrt{c} d^2 - \sqrt{a} e^2) (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d^2 e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+c x^4}} \right) /
\end{aligned}$$

Result (type 4, 531 leaves):

$$\begin{aligned}
& \frac{1}{e^4 \sqrt{a + c x^4}} \left(-2 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \right. \\
& \frac{2 \sqrt{c} \left(i \sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} + \\
& 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] - \frac{1}{\sqrt{c d^4 + a e^4} (d + e x)} e \left(a e^2 \sqrt{c d^4 + a e^4} + \right. \\
& c e^2 \sqrt{c d^4 + a e^4} x^4 + c d^3 (d + e x) \sqrt{a + c x^4} \operatorname{Log}\left[-d^2 + e^2 x^2\right] + \sqrt{c} d \sqrt{c d^4 + a e^4} (d + e x) \sqrt{a + c x^4} \operatorname{Log}\left[c x^2 + \sqrt{c} \sqrt{a + c x^4}\right] - \\
& \left. \left. c d^4 \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] - c d^3 e x \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right]\right) \right)
\end{aligned}$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 295 leaves, 9 steps):

$$\begin{aligned}
& \frac{e^3 \sqrt{a + c x^4}}{2 c} + \frac{3 d e^2 x \sqrt{a + c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{3 d^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right]}{2 \sqrt{c}} - \frac{3 a^{1/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{a + c x^4}} + \\
& \frac{d (\sqrt{c} d^2 + 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{3/4} \sqrt{a + c x^4}}
\end{aligned}$$

Result (type 4, 240 leaves):

$$\frac{1}{2 \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} c \sqrt{a + c x^4}} \left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} e \left(e^2 (a + c x^4) + 3 \sqrt{c} d^2 \sqrt{a + c x^4} \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right] \right) + 6 \sqrt{a} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x \right], -1 \right] - 2 \sqrt{c} d \left(i \sqrt{c} d^2 + 3 \sqrt{a} e^2 \right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x \right], -1 \right] \right)$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^2}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 263 leaves, 8 steps):

$$\frac{e^2 x \sqrt{a + c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{d e \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right]}{\sqrt{c}} - \frac{a^{1/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{c^{3/4} \sqrt{a + c x^4}} + \frac{a^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2 \right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 c^{3/4} \sqrt{a + c x^4}}$$

Result (type 4, 204 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} \sqrt{c} \sqrt{a + c x^4}} \left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} d e \sqrt{a + c x^4} \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right] + \sqrt{a} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x \right], -1 \right] - \left(i \sqrt{c} d^2 + \sqrt{a} e^2 \right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x \right], -1 \right] \right)$$

Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d + e x}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$\frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right]}{2 \sqrt{c}} + \frac{d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 107 leaves):

$$\frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right]}{2 \sqrt{c}} - \frac{i d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}}$$

Problem 215: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{(\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}}$$

Problem 216: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x) \sqrt{a + c x^4}} dx$$

Optimal (type 4, 405 leaves, 7 steps):

$$\frac{e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right] - e \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right] + \frac{c^{1/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 \sqrt{-c d^4 - a e^4} - 2 \sqrt{c d^4 + a e^4} + 2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a + c x^4}} - \frac{(\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a + c x^4}}$$

Result (type 4, 200 leaves):

$$\left(\sqrt{1 + \frac{c x^4}{a}} \right) \left(-2 (-1)^{1/4} a^{1/4} \sqrt{1 + \frac{c d^4}{a e^4}} e \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + c^{1/4} d \operatorname{Log}\left[\frac{-d^2 + e^2 x^2}{c d^2 x^2 + a e^2 \left(1 + \sqrt{1 + \frac{c d^4}{a e^4}} \sqrt{1 + \frac{c x^4}{a}}\right)}\right] \right) \Bigg/ 2 \left(c^{1/4} d \sqrt{1 + \frac{c d^4}{a e^4}} e \sqrt{a + c x^4} \right)$$

Problem 217: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x)^2 \sqrt{a + c x^4}} dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$\begin{aligned}
& - \frac{e^3 \sqrt{a+c x^4}}{(c d^4+a e^4)(d+e x)} + \frac{\sqrt{c} e^2 x \sqrt{a+c x^4}}{(c d^4+a e^4)(\sqrt{a}+\sqrt{c} x^2)} - \frac{c d^3 e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4-a e^4} x}{d e \sqrt{a+c x^4}}\right]}{(-c d^4-a e^4)^{3/2}} - \frac{c d^3 e \operatorname{ArcTanh}\left[\frac{a e^2+c d^2 x^2}{\sqrt{c d^4+a e^4} \sqrt{a+c x^4}}\right]}{(c d^4+a e^4)^{3/2}} - \\
& \frac{a^{1/4} c^{1/4} e^2 (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{(c d^4+a e^4) \sqrt{a+c x^4}} + \frac{c^{1/4} (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2+\sqrt{a} e^2) \sqrt{a+c x^4}} - \\
& \frac{c^{3/4} d^2 (\sqrt{c} d^2-\sqrt{a} e^2) (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2+\sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2+\sqrt{a} e^2) (c d^4+a e^4) \sqrt{a+c x^4}}
\end{aligned}$$

Result (type 4, 462 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} (c d^4+a e^4)^{3/2} (d+e x) \sqrt{a+c x^4}} \left(\sqrt{a} \sqrt{c} e^2 \sqrt{c d^4+a e^4} (d+e x) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& i \sqrt{c} (\sqrt{c} d^2+i \sqrt{a} e^2) \sqrt{c d^4+a e^4} (d+e x) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \\
& \left. \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(e^3 \sqrt{c d^4+a e^4} (a+c x^4) + 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{c d^4+a e^4} (d+e x) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] - \right. \right. \\
& \left. \left. c d^3 e (d+e x) \sqrt{a+c x^4} \operatorname{Log}\left[-d^2+e^2 x^2\right] + c d^3 e (d+e x) \sqrt{a+c x^4} \operatorname{Log}\left[a e^2+c d^2 x^2+\sqrt{c d^4+a e^4} \sqrt{a+c x^4}\right] \right) \right)
\end{aligned}$$

Problem 218: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^3 \sqrt{a+c x^4}} dx$$

Optimal (type 4, 659 leaves, 12 steps):

$$\begin{aligned}
& - \frac{e^3 \sqrt{a + c x^4}}{2 (c d^4 + a e^4) (d + e x)^2} - \frac{3 c d^3 e^3 \sqrt{a + c x^4}}{(c d^4 + a e^4)^2 (d + e x)} + \frac{3 c^{3/2} d^3 e^2 x \sqrt{a + c x^4}}{(c d^4 + a e^4)^2 (\sqrt{a} + \sqrt{c} x^2)} + \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right]}{2 (-c d^4 - a e^4)^{5/2}} - \\
& \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right]}{2 (c d^4 + a e^4)^{5/2}} - \frac{3 a^{1/4} c^{5/4} d^3 e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{(c d^4 + a e^4)^2 \sqrt{a + c x^4}} + \\
& \frac{c^{3/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (c d^4 + a e^4) \sqrt{a + c x^4}} - \\
& \left(\frac{3 c^{3/4} d (\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} (c d^4 + a e^4)^2 \sqrt{a + c x^4}} \right) /
\end{aligned}$$

Result (type 4, 884 leaves):

$$\begin{aligned}
& \frac{1}{2 (c d^4 + a e^4)^{5/2} (d + e x)^2 \sqrt{a + c x^4}} \\
& \left(-e^3 (c d^4 + a e^4)^{3/2} (a + c x^4) - 6 c d^3 e^3 \sqrt{c d^4 + a e^4} (d + e x) (a + c x^4) - 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \right. \\
& \quad \text{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \frac{4 i c^2 d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} + \\
& \quad 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \\
& \quad \frac{2 i a c d e^4 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} - \\
& \quad 6 (-1)^{1/4} a^{1/4} c^{7/4} d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + \\
& \quad 6 (-1)^{1/4} a^{5/4} c^{3/4} d e^4 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + \\
& \quad 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \operatorname{Log}[-d^2 + e^2 x^2] - 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \operatorname{Log}[-d^2 + e^2 x^2] - 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \\
& \quad \left. \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] + 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] \right)
\end{aligned}$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 298 leaves, 4 steps):

$$\frac{3 d e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} - \frac{a e^3 - c x (d^3 + 3 d^2 e x + 3 d e^2 x^2)}{2 a c \sqrt{a + c x^4}} + \frac{3 d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} +$$

$$\frac{d (\sqrt{c} d^2 - 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a + c x^4}}$$

Result (type 4, 215 leaves):

$$\frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a + c x^4}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} (-a e^3 + c d x (d^2 + 3 d e x + 3 e^2 x^2)) - 3 \sqrt{a} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right.$$

$$\left. \sqrt{c} d (-i \sqrt{c} d^2 + 3 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)$$

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^2}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$\frac{x (d + e x)^2}{2 a \sqrt{a + c x^4}} - \frac{e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} +$$

$$\frac{(\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a + c x^4}}$$

Result (type 4, 188 leaves):

$$\frac{1}{2 a^{3/2} \left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{a+c x^4}} i \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} x (d+e x)^2 - \sqrt{a} e^2 \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + (-i \sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)$$

Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x}{(a+c x^4)^{3/2}} dx$$

Optimal (type 4, 114 leaves, 3 steps):

$$\frac{x (d+e x)}{2 a \sqrt{a+c x^4}} + \frac{d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 90 leaves):

$$x (d+e x) - \frac{i d \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$2 a \sqrt{a+c x^4}$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+c x^4)^{3/2}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{x}{2 a \sqrt{a+c x^4}} + \frac{(\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 102 leaves):

$$\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}} x - i} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{2 a \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}}$$

Problem 223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x) (a + c x^4)^{3/2}} dx$$

Optimal (type 4, 818 leaves, 14 steps):

$$\begin{aligned} & \frac{e (a e^2 - c d^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} + \frac{c d x (d^2 + e^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} - \frac{\sqrt{c} d e^2 x \sqrt{a + c x^4}}{2 a (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2)} - \frac{e^5 \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right]}{2 (-c d^4 - a e^4)^{3/2}} \\ & \frac{e^5 \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right]}{2 (c d^4 + a e^4)^{3/2}} + \frac{c^{1/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} (c d^4 + a e^4) \sqrt{a + c x^4}} + \\ & \frac{c^{1/4} d (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} (c d^4 + a e^4) \sqrt{a + c x^4}} + \\ & \frac{c^{1/4} d e^4 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4}} - \\ & \frac{e^4 (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4}} \end{aligned}$$

Result (type 4, 464 leaves):

$$\frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^{1/4} d (c d^4 + a e^4)^{3/2} \sqrt{a + c x^4}} \left(-\sqrt{a} c^{3/4} d^2 e^2 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\ \left. c^{3/4} d^2 \left(-i \sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\ \left. \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(-2 (-1)^{1/4} a^{5/4} e^4 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + c^{1/4} d \right. \right. \\ \left. \left. \left(\sqrt{c d^4 + a e^4} (a e^3 + c d x (d^2 - d e x + e^2 x^2)) + a e^5 \sqrt{a + c x^4} \text{Log}[-d^2 + e^2 x^2] - a e^5 \sqrt{a + c x^4} \text{Log}[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}]\right) \right) \right)$$

Problem 224: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^n}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\frac{(c + d x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}\right]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (1 + n)} - \frac{(c + d x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}\right]}{4 b^{3/4} \left(b^{1/4} c + \sqrt{-\sqrt{-a}} d\right) (1 + n)} \\ - \frac{(c + d x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - (-a)^{1/4} d}\right]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (1 + n)} - \frac{(c + d x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + (-a)^{1/4} d}\right]}{4 b^{3/4} \left(b^{1/4} c + (-a)^{1/4} d\right) (1 + n)}$$

Result (type 7, 526 leaves):

$$\frac{1}{4 b n} (c + d x)^n \left(c^3 \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] - \right.$$

$$3 c^2 \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] +$$

$$3 c \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] -$$

$$\left. \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}\right] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] \right)$$

Problem 225: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^{1+n}}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}\right]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}\right]}{4 b^{3/4} \left(b^{1/4} c + \sqrt{-\sqrt{-a}} d\right) (2 + n)}$$

$$\frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - (-a)^{1/4} d}\right]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + (-a)^{1/4} d}\right]}{4 b^{3/4} \left(b^{1/4} c + (-a)^{1/4} d\right) (2 + n)}$$

Result (type 7, 691 leaves):

$$\frac{1}{4 b^2 n (1+n)} (c+d x)^n \left((b c^4 + a d^4) (1+n) \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}\right] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] - b \left(-4 c n - 4 d n x + 3 c^3 (1+n) \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}\right] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] - 3 c^2 (1+n) \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}\right] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] + c \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}\right] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] + c n \operatorname{RootSum}\left[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}\right] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&\right] \right)$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c+d x+e x^2) \sqrt{a+b x^4}} dx$$

Optimal (type 4, 1605 leaves, 16 steps):

$$\frac{e^2 \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{-b d^4+4 b c d^2 e-2 b c^2 e^2-2 a e^4-b d \sqrt{d^2-4 c e}} (d^2-2 c e) x}{e\left(d+\sqrt{d^2-4 c e}\right) \sqrt{a+b x^4}}\right]}{\sqrt{2} \sqrt{d^2-4 c e} \sqrt{-2 a e^4-b\left(d^4-4 c d^2 e+2 c^2 e^2+d^3 \sqrt{d^2-4 c e}-2 c d e \sqrt{d^2-4 c e}\right)}} + \frac{e^2 \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{-b d^4+4 b c d^2 e-2 b c^2 e^2-2 a e^4+b d \sqrt{d^2-4 c e}} (d^2-2 c e) x}{e\left(d-\sqrt{d^2-4 c e}\right) \sqrt{a+b x^4}}\right]}{\sqrt{2} \sqrt{d^2-4 c e} \sqrt{-2 a e^4-b\left(d^4-4 c d^2 e+2 c^2 e^2-d^3 \sqrt{d^2-4 c e}+2 c d e \sqrt{d^2-4 c e}\right)}}$$

$$\begin{aligned}
& \frac{e^2 \operatorname{ArcTanh}\left[\frac{4ae^2+b(d-\sqrt{d^2-4ce})^2x^2}{2\sqrt{2}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4-bd\sqrt{d^2-4ce}(d^2-2ce)}\sqrt{a+bx^4}}\right]}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4-bd\sqrt{d^2-4ce}(d^2-2ce)}} + \\
& \frac{e^2 \operatorname{ArcTanh}\left[\frac{4ae^2+b(d+\sqrt{d^2-4ce})^2x^2}{2\sqrt{2}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4+bd\sqrt{d^2-4ce}(d^2-2ce)}\sqrt{a+bx^4}}\right]}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4+bd\sqrt{d^2-4ce}(d^2-2ce)}} + \\
& \frac{b^{1/4}e(d-\sqrt{d^2-4ce})(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{1/4}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce-d\sqrt{d^2-4ce})\right)\sqrt{a+bx^4}} - \\
& \frac{b^{1/4}e(d+\sqrt{d^2-4ce})(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{1/4}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce+d\sqrt{d^2-4ce})\right)\sqrt{a+bx^4}} + \left(e\left(2\sqrt{a}e^2-\sqrt{b}(d^2-2ce-d\sqrt{d^2-4ce})\right)\right. \\
& \left. (\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticPi}\left[\frac{\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce-d\sqrt{d^2-4ce})\right)^2}{4\sqrt{a}\sqrt{b}e^2(d-\sqrt{d^2-4ce})^2}, 2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]\right) / \\
& \left(2a^{1/4}b^{1/4}\sqrt{d^2-4ce}(d-\sqrt{d^2-4ce})\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce-d\sqrt{d^2-4ce})\right)\sqrt{a+bx^4}\right) - \left(e\left(2\sqrt{a}e^2-\sqrt{b}(d^2-2ce+d\sqrt{d^2-4ce})\right)\right) \\
& \left. (\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticPi}\left[\frac{\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce+d\sqrt{d^2-4ce})\right)^2}{4\sqrt{a}\sqrt{b}e^2(d+\sqrt{d^2-4ce})^2}, 2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]\right) / \\
& \left(2a^{1/4}b^{1/4}\sqrt{d^2-4ce}(d+\sqrt{d^2-4ce})\left(2\sqrt{a}e^2+\sqrt{b}(d^2-2ce+d\sqrt{d^2-4ce})\right)\sqrt{a+bx^4}\right)
\end{aligned}$$

Result (type 4, 653 leaves):

$$\begin{aligned}
& - \left(\left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} x\right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \left(i \sqrt{a} + \sqrt{b} x^2 \right) \right. \right. \\
& \left. \left(b^{1/4} \left(-\sqrt{b} c + (-1)^{1/4} a^{1/4} b^{1/4} d - i \sqrt{a} e \right) \sqrt{d^2 - 4 c e} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} x\right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] + \right. \right. \\
& \left. \left. (-1)^{1/4} a^{1/4} \left(-\left(-2 i \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \operatorname{EllipticPi} \left[\frac{2 (-1)^{3/4} a^{1/4} e - i b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)}{2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)} \right], \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} x\right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] - \left(2 i \sqrt{a} e^2 + \sqrt{b} \left(-d^2 + 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[-\frac{i \left(2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right) \right)}{-2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right)} \right], \operatorname{ArcSin} \left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} x\right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] \right) \right) / \\
& \left(a^{1/4} \sqrt{d^2 - 4 c e} \left(b c^2 - a e^2 - i \sqrt{a} \sqrt{b} \left(d^2 - 2 c e \right) \right) \sqrt{\frac{i \sqrt{a} + \sqrt{b} x^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} x \right)^2}} \sqrt{a + b x^4} \right)
\end{aligned}$$

Problem 253: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{2}{3} \left(c \sqrt{a + b x^2} \right)^{3/2} + \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTan} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}} - \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTanh} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{2 c^2 \left(a + b x^2 - 3 a \left(1 + \frac{a}{b x^2} \right)^{1/4} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^2} \right] \right)}{3 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 254: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sqrt{a + b x^2})^{3/2}}{x^3} dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{(c \sqrt{a + b x^2})^{3/2}}{2 x^2} + \frac{3 b (c \sqrt{a + b x^2})^{3/2} \operatorname{ArcTan}\left[\left(1 + \frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1 + \frac{b x^2}{a}\right)^{3/4}} - \frac{3 b (c \sqrt{a + b x^2})^{3/2} \operatorname{ArcTanh}\left[\left(1 + \frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 73 leaves):

$$-\frac{c^2 (a + b x^2 + 3 b \left(1 + \frac{a}{b x^2}\right)^{1/4} x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^2}\right])}{2 x^2 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 255: Result unnecessarily involves higher level functions.

$$\int x^2 (c \sqrt{a + b x^2})^{3/2} dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{2 a x (c \sqrt{a + b x^2})^{3/2}}{15 b} + \frac{2}{9} x^3 (c \sqrt{a + b x^2})^{3/2} - \frac{4 a^2 x (c \sqrt{a + b x^2})^{3/2}}{15 b (a + b x^2)} + \frac{4 a^{3/2} (c \sqrt{a + b x^2})^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{15 b^{3/2} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 88 leaves):

$$\frac{2 c^2 \left(3 a^2 x + 8 a b x^3 + 5 b^2 x^5 - 3 a^2 x \left(1 + \frac{b x^2}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{45 b \sqrt{c \sqrt{a + b x^2}}}$$

Problem 256: Result unnecessarily involves higher level functions.

$$\int (c \sqrt{a + b x^2})^{3/2} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$\frac{2}{5} x \left(c \sqrt{a + b x^2} \right)^{3/2} + \frac{6 a x \left(c \sqrt{a + b x^2} \right)^{3/2}}{5 (a + b x^2)} - \frac{6 \sqrt{a} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{5 \sqrt{b} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 71 leaves):

$$\frac{c^2 x \left(2 (a + b x^2) + 3 a \left(1 + \frac{b x^2}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{5 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 257: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x^2} dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$-\frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x} + \frac{3 b x \left(c \sqrt{a + b x^2} \right)^{3/2}}{a + b x^2} - \frac{3 \sqrt{b} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{\sqrt{a} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 76 leaves):

$$-\frac{c^2 \left(2 (a + b x^2) - 3 b x^2 \left(1 + \frac{b x^2}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{2 x \sqrt{c \sqrt{a + b x^2}}}$$

Problem 258: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x^4} dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$-\frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{3 x^3} - \frac{b \left(c \sqrt{a + b x^2} \right)^{3/2}}{2 a x} + \frac{b^2 x \left(c \sqrt{a + b x^2} \right)^{3/2}}{2 a (a + b x^2)} - \frac{b^{3/2} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{2 a^{3/2} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 92 leaves):

$$-\frac{c^2 \left(4 a^2 + 10 a b x^2 + 6 b^2 x^4 - 3 b^2 x^4 \left(1 + \frac{b x^2}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a} \right] \right)}{12 a x^3 \sqrt{c} \sqrt{a + b x^2}}$$

Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{(1-x^2)(3+x^2)} \, dx$$

Optimal (type 4, 48 leaves, 6 steps):

$$\frac{1}{3} x \sqrt{3-2x^2-x^4} - \frac{2 \text{EllipticE}[\text{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}} + \frac{4 \text{EllipticF}[\text{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 59 leaves):

$$\frac{1}{3} \left(x \sqrt{3-2x^2-x^4} - 2 i \text{EllipticE} \left[i \text{ArcSinh} \left[\frac{x}{\sqrt{3}} \right], -3 \right] - 4 i \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{x}{\sqrt{3}} \right], -3 \right] \right)$$

Problem 262: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx$$

Optimal (type 4, 12 leaves, 3 steps):

$$\frac{\text{EllipticF}[\text{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 18 leaves):

$$-i \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{x}{\sqrt{3}} \right], -3 \right]$$

Problem 266: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} \, dx$$

Optimal (type 3, 112 leaves, 5 steps):

$$-\frac{\sqrt{a} \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{\sqrt{c}} + \frac{\sqrt{b} \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right]}{\sqrt{d}}$$

Result (type 6, 245 leaves):

$$\left(5 a (b c - a d) (a + b x^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] \right) /$$

$$\left(3 b x^2 \left(5 a (b c - a d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] - \right. \right.$$

$$\left. \left. (a + b x^2) \left((-2 b c + 2 a d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] + a d \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] \right) \right) \right)$$

Problem 267: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\frac{(b c - a d) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2 c \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)} - \frac{(b c - a d) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{2 \sqrt{a} c^{3/2}}$$

Result (type 6, 190 leaves):

$$\frac{1}{2 c x^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(-c - d x^2 + \left(2 b d (b c - a d) x^4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left((a + b x^2) \right. \right.$$

$$\left. \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) \right)$$

Problem 268: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5} dx$$

Optimal (type 3, 208 leaves, 5 steps):

$$-\frac{(bc-ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{8a^{3/2} c^{5/2}}$$

Result (type 6, 224 leaves):

$$-\frac{1}{8ac^2 x^4} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left((c+dx^2)(2ac+bcx^2-3adx^2) + \left(2bd(b^2c^2+2abcd-3a^2d^2)x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right]\right) / \left((a+bx^2)\right. \right. \\ \left. \left. \left(-4bdx^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right] + bc \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right] + ad \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right]\right)\right) \right)$$

Problem 269: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$\frac{(bc-ad)^2 (bc+3ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(b^2c^2+2abcd-11a^2d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} + \\ \frac{(bc-ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)(b^2c^2+2abcd+5a^2d^2) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{16a^{5/2} c^{7/2}}$$

Result (type 6, 272 leaves):

$$\frac{1}{48 a^2 c^3 x^6} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((c + d x^2) (3 b^2 c^2 x^4 - 2 a b c x^2 (c - 2 d x^2) + a^2 (-8 c^2 + 10 c d x^2 - 15 d^2 x^4)) + \right. \\ \left. \left(6 b d (b^3 c^3 + a b^2 c^2 d + 3 a^2 b c d^2 - 5 a^3 d^3) x^8 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left((a + b x^2) \right. \right. \\ \left. \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) \right)$$

Problem 270: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} dx$$

Optimal (type 4, 357 leaves, 7 steps):

$$\frac{(8 b^2 c^2 - 3 a b c d - 2 a^2 d^2) x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{15 b^2 d^2} - \frac{(4 b c - a d) x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{15 b d^2} + \frac{x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d} - \\ \frac{\sqrt{c} (8 b^2 c^2 - 3 a b c d - 2 a^2 d^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 b^2 d^{5/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}} + \frac{c^{3/2} (4 b c - a d) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 b d^{5/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}$$

Result (type 4, 255 leaves):

$$\frac{1}{15 b \sqrt{\frac{b}{a}} d^3 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \\ \left(\sqrt{\frac{b}{a}} d x (a + b x^2) (c + d x^2) (-4 b c + a d + 3 b d x^2) + i c (-8 b^2 c^2 + 3 a b c d + 2 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - \right. \\ \left. i c (-8 b^2 c^2 + 7 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 271: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$$

Optimal (type 4, 266 leaves, 6 steps):

$$\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}} + x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3bd} + \frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right] - c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Result (type 4, 208 leaves):

$$\frac{1}{3\sqrt{\frac{b}{a}d^2(a+bx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{\frac{b}{a}} dx(a+bx^2)(c+dx^2) - ic(-2bc+ad) \sqrt{1+\frac{bx^2}{a}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] + 2ic(-bc+ad) \sqrt{1+\frac{bx^2}{a}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] \right)$$

Problem 274: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\frac{d (b c - 2 a d) x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{3 a c^2} - \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{3 c x^3} - \frac{(b c - 2 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{3 a c^2 x} -$$

$$\frac{\sqrt{d} (b c - 2 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a c^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \frac{b \sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a \sqrt{c} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}$$

Result (type 4, 238 leaves):

$$-\frac{1}{3 b c^2 x^3 (a+b x^2)} \sqrt{\frac{b}{a}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}$$

$$\left(\sqrt{\frac{b}{a}} (a+b x^2) (c+d x^2) (b c x^2 + a (c-2 d x^2)) - i b c (-b c + 2 a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \right.$$

$$\left. i b c (-b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 275: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{x^6} dx$$

Optimal (type 4, 424 leaves, 8 steps):

$$\begin{aligned}
& \frac{d (2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) x \sqrt{\frac{e (a+b x^2)}{c+d x^2}} - \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{15 a^2 c^3} - \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{5 c x^5} \\
& \frac{(b c - 4 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{15 a c^2 x^3} + \frac{(2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c+d x^2)}{15 a^2 c^3 x} \\
& \frac{\sqrt{d} (2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right] - b \sqrt{d} (b c - 4 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 a^2 c^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \frac{b \sqrt{d} (b c - 4 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 a^2 c^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 302 leaves):

$$\begin{aligned}
& - \frac{1}{15 a^2 \sqrt{\frac{b}{a}} c^3 x^5 (a+b x^2)} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \left(\sqrt{\frac{b}{a}} (a+b x^2) (c+d x^2) (-2 b^2 c^2 x^4 + a b c x^2 (c-3 d x^2) + a^2 (3 c^2 - 4 c d x^2 + 8 d^2 x^4)) + \right. \\
& \quad \left. i b c (-2 b^2 c^2 - 3 a b c d + 8 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - \right. \\
& \quad \left. 2 i b c (-b^2 c^2 - a b c d + 2 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)
\end{aligned}$$

Problem 279: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x} dx$$

Optimal (type 3, 151 leaves, 6 steps):

$$\frac{(b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d} - \frac{a^{3/2} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{c^{3/2}} + \frac{b^{3/2} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{b} \sqrt{e}}\right]}{d^{3/2}}$$

Result (type 6, 330 leaves):

$$\frac{1}{c d (a + b x^2)} e^{\sqrt{\frac{e (a + b x^2)}{c + d x^2}}} \left((-b c + a d) (a + b x^2) + \left(2 a^2 b d^2 x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) / \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] - \left(2 a b^2 c^2 x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) \right) / \left(-4 a c \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + x^2 \left(a d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) \right)$$

Problem 280: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}}{x^3} dx$$

Optimal (type 3, 165 leaves, 5 steps):

$$\frac{3 (b c - a d) e^{\sqrt{\frac{e (a + b x^2)}{c + d x^2}}}}{2 c^2} + \frac{(b c - a d) \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}}{2 c \left(a - \frac{c (a + b x^2)}{c + d x^2} \right)} - \frac{3 \sqrt{a} (b c - a d) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{2 c^{5/2}}$$

Result (type 6, 210 leaves):

$$\frac{1}{2 c^2 x^2 (a + b x^2)} e^{\sqrt{\frac{e (a + b x^2)}{c + d x^2}}} \left(-(a + b x^2) (-2 b c x^2 + a (c + 3 d x^2)) + \left(6 a b d (b c - a d) x^4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) / \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right)$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}}{x^5} dx$$

Optimal (type 3, 256 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d (b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c^3} - \frac{a (b c - a d)^2 e^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{4 c^3 \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)^2} + \\
& \frac{(5 b c - 9 a d) (b c - a d) e^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{8 c^3 \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)} - \frac{3 (b c - 5 a d) (b c - a d) e^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{8 \sqrt{a} c^{7/2}}
\end{aligned}$$

Result (type 6, 245 leaves):

$$\begin{aligned}
& \frac{1}{8 c^3 x^4 (a+b x^2)} e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \\
& \left((a+b x^2) (-b c x^2 (5 c+13 d x^2) + a (-2 c^2+5 c d x^2+15 d^2 x^4)) + \left(6 b d (b^2 c^2-6 a b c d+5 a^2 d^2) x^6 \operatorname{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] \right) / \right. \\
& \left. \left(-4 b d x^2 \operatorname{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] + b c \operatorname{AppellF1} \left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] + a d \operatorname{AppellF1} \left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] \right) \right)
\end{aligned}$$

Problem 282: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2} \right)^{3/2}}{x^7} dx$$

Optimal (type 3, 366 leaves, 7 steps):

$$\begin{aligned}
& \frac{d^2 (b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c^4} + \frac{(b c - a d)^3 e^2 \left(\frac{e (a+b x^2)}{c+d x^2} \right)^{5/2}}{6 a c^2 \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)^3} + \frac{(b c - a d)^2 (b c + 11 a d) e^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{24 c^4 \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)^2} - \\
& \frac{(b c - a d) (5 b^2 c^2 + 50 a b c d - 79 a^2 d^2) e^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{48 a c^4 \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)} + \frac{(b c - a d) (b^2 c^2 + 10 a b c d - 35 a^2 d^2) e^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{16 a^{3/2} c^{9/2}}
\end{aligned}$$

Result (type 6, 305 leaves):

$$\begin{aligned}
& - \frac{1}{48 a c^4 x^6 (a + b x^2)} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((a + b x^2) (3 b^2 c^2 x^4 (c + d x^2) + 2 a b c x^2 (7 c^2 - 19 c d x^2 - 50 d^2 x^4) + a^2 (8 c^3 - 14 c^2 d x^2 + 35 c d^2 x^4 + 105 d^3 x^6)) + \right. \\
& \left. \left(6 b d (b^3 c^3 + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3) x^8 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \right. \\
& \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)
\end{aligned}$$

Problem 283: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 391 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\left(16 a c - \frac{16 b c^2}{d} - \frac{a^2 d}{b} \right) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} - e x^3 (a + b x^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} - \frac{(8 b c - 7 a d) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d^3} + \\
& \frac{6 b e x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d^2} - \frac{\sqrt{c} (16 b^2 c^2 - 16 a b c d + a^2 d^2) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 b d^{7/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}} + \\
& \frac{c^{3/2} (8 b c - 7 a d) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 d^{7/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 290 leaves):

$$\frac{1}{5 \sqrt{\frac{b}{a} d^4 (a + b x^2)}} e^{\sqrt{\frac{e (a + b x^2)}{c + d x^2}}}$$

$$\left(\sqrt{\frac{b}{a}} d x (a^2 d (7 c + 2 d x^2) + b^2 x^2 (-8 c^2 - 2 c d x^2 + d^2 x^4) + a b (-8 c^2 + 5 c d x^2 + 3 d^2 x^4)) - i c (16 b^2 c^2 - 16 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right.$$

$$\left. \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + 8 i c (2 b^2 c^2 - 3 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 284: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 310 leaves, 7 steps):

$$\frac{(8 b c - 7 a d) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{3 d^2} - \frac{e x (a + b x^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{d} + \frac{4 b e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{3 d^2} +$$

$$\frac{\sqrt{c} (8 b c - 7 a d) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 d^{5/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}} - \frac{\sqrt{c} (4 b c - 3 a d) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 d^{5/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}$$

Result (type 4, 235 leaves):

$$\begin{aligned}
& - \frac{1}{3 \sqrt{\frac{b}{a}} d^3 (a + b x^2)} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} d x (a + b x^2) (3 a d - b (4 c + d x^2)) + i b c (-8 b c + 7 a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \right. \\
& \left. i (8 b^2 c^2 - 11 a b c d + 3 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)
\end{aligned}$$

Problem 285: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 262 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(b c - a d) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{c d} + \frac{(2 b c - a d) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{c d} \\
& \frac{(2 b c - a d) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right] + b \sqrt{c} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{\sqrt{c} d^{3/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} + d^{3/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 206 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{b}{a}} c d^2 (a + b x^2)} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(i b c (-2 b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \right. \\
& \left. (-b c + a d) \left(\sqrt{\frac{b}{a}} d x (a + b x^2) - 2 i b c \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right) \right)
\end{aligned}$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 307 leaves, 7 steps):

$$\begin{aligned} & -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \\ & \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \end{aligned}$$

Result (type 4, 228 leaves):

$$\begin{aligned} & -\frac{1}{\sqrt{\frac{b}{a}c^2dx(a+bx^2)}} \\ & e\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{\frac{b}{a}d(a+bx^2)(ac-bcx^2+2adx^2)} + i b c (-bc+2ad) x \sqrt{1+\frac{bx^2}{a}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] - \right. \\ & \left. i b c (-bc+ad) x \sqrt{1+\frac{bx^2}{a}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] \right) \end{aligned}$$

Problem 287: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 383 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(bc - ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc - 8ad) e x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc - 4ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{3c^2 dx^3} - \frac{(7bc - 8ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{3c^3 x} \\
& \frac{\sqrt{d} (7bc - 8ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{3c^{5/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b(3bc - 4ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{3ac^{3/2} \sqrt{d} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 275 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{\frac{b}{a}} c^3 x^3 (a + bx^2)} \\
& e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(- \sqrt{\frac{b}{a}} (b^2 c x^4 (4c + 7dx^2) + a^2 (c^2 - 4cdx^2 - 8d^2 x^4) + abx^2 (5c^2 + 3cdx^2 - 8d^2 x^4)) + i bc (-7bc + 8ad) x^3 \sqrt{1 + \frac{bx^2}{a}} \right. \\
& \left. \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] - 4 i bc (-bc + ad) x^3 \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] \right)
\end{aligned}$$

Problem 288: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6} dx$$

Optimal (type 4, 480 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(bc - ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2 - 16abcd + 16a^2d^2) e x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc - 6ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{5c^2dx^5} \\
& - \frac{(7bc - 8ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{5c^3x^3} - \frac{(b^2c^2 - 16abcd + 16a^2d^2) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{5ac^4x} \\
& - \frac{\sqrt{d} (b^2c^2 - 16abcd + 16a^2d^2) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{5ac^{7/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\
& - \frac{b\sqrt{d} (7bc - 8ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{5ac^{5/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 357 leaves):

$$\begin{aligned}
& - \frac{1}{5bc^4x^5(a+bx^2)} \sqrt{\frac{b}{a}} e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{\frac{b}{a}} \right. \\
& \quad \left. (b^3c^2x^6(c+dx^2) + ab^2cx^4(3c^2 - 8cdx^2 - 16d^2x^4) + a^2bx^2(3c^3 - 11c^2dx^2 - 8cd^2x^4 + 16d^3x^6) + a^3(c^3 - 2c^2dx^2 + 8cd^2x^4 + 16d^3x^6)) + \right. \\
& \quad \left. i b c (b^2c^2 - 16abcd + 16a^2d^2) x^5 \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] - \right. \\
& \quad \left. i b c (b^2c^2 - 9abcd + 8a^2d^2) x^5 \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] \right)
\end{aligned}$$

Problem 299: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 3, 112 leaves, 5 steps):

$$-\frac{\sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{\sqrt{a} \sqrt{e}} + \frac{\sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right]}{\sqrt{b} \sqrt{e}}$$

Result (type 6, 244 leaves):

$$\left(3 a (b c - a d) (a + b x^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{d(a+bx^2)}{-bc+ad}, 1 + \frac{bx^2}{a}\right] \right) /$$

$$\left(b x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(3 a (b c - a d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{d(a+bx^2)}{-bc+ad}, 1 + \frac{bx^2}{a}\right] - \right.$$

$$\left. (a + b x^2) \left((-2 b c + 2 a d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \frac{d(a+bx^2)}{-bc+ad}, 1 + \frac{bx^2}{a}\right] - a d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d(a+bx^2)}{-bc+ad}, 1 + \frac{bx^2}{a}\right] \right) \right) \right)$$

Problem 300: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\frac{(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a \left(a e - \frac{c e(a+bx^2)}{c+dx^2} \right)} + \frac{(bc - ad) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{2 a^{3/2} \sqrt{c} \sqrt{e}}$$

Result (type 6, 190 leaves):

$$-a - b x^2 + \frac{2 b d (-b c + a d) x^4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right]}{(c + d x^2) \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right)}$$

$$2 a x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 3, 218 leaves, 5 steps):

$$\frac{(bc-ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(bc-ad)(3bc+ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(bc-ad)(3bc+ad) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right]}{8a^{5/2} c^{3/2} \sqrt{e}}$$

Result (type 6, 224 leaves):

$$-\frac{1}{8a^2c x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \left((a+bx^2)(2ac-3bcx^2+adx^2) + (2bd(-3b^2c^2+2abcd+a^2d^2)x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right]) \right) / \left((c+dx^2) \right. \\ \left. \left(-4bdx^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right] + bc \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right] + ad \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{bx^2}, -\frac{c}{dx^2}\right] \right) \right)$$

Problem 302: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 4, 403 leaves, 7 steps):

$$\frac{(bc-4ad)x(a+bx^2)}{15b^2d \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)x(a+bx^2)}{15b^3d \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \\ \frac{\sqrt{c}(2b^2c^2+3abcd-8a^2d^2)(a+bx^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{15b^3d^{3/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{c^{3/2}(bc-4ad)(a+bx^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{15b^2d^{3/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Result (type 4, 258 leaves):

$$\left(-\sqrt{\frac{b}{a}} dx (a + bx^2) (c + dx^2) (4ad - b(c + 3dx^2)) - ic(-2b^2c^2 - 3abcd + 8a^2d^2) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] + \right. \\ \left. 2ic(-b^2c^2 - abcd + 2a^2d^2) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] \right) / \left(15a^2 \left(\frac{b}{a}\right)^{5/2} d^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2) \right)$$

Problem 303: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$\frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}(bc-2ad)(a+bx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right] - c^{3/2}(a+bx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2) - 3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Result (type 4, 212 leaves):

$$\left(\sqrt{\frac{b}{a}} dx (a + bx^2) (c + dx^2) + ic(-bc + 2ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] - \right. \\ \left. ic(-bc + ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}\right] \right) / \left(3b\sqrt{\frac{b}{a}} d \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2) \right)$$

Problem 306: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 4, 372 leaves, 7 steps):

$$-\frac{a + b x^2}{3 a x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} + \frac{(2 b c - a d) (a + b x^2)}{3 a^2 c x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} - \frac{d (2 b c - a d) x (a + b x^2)}{3 a^2 c \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} +$$

$$\frac{\sqrt{d} (2 b c - a d) (a + b x^2) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a^2 \sqrt{c} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} - \frac{b \sqrt{c} \sqrt{d} (a + b x^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a^2 \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}$$

Result (type 4, 238 leaves):

$$\left(-\sqrt{\frac{b}{a}} (a + b x^2) (c + d x^2) (-2 b c x^2 + a (c + d x^2)) - i b c (-2 b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \right.$$

$$\left. 2 i b c (-b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right) / \left(3 a^2 \sqrt{\frac{b}{a}} c x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2) \right)$$

Problem 310: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 152 leaves, 6 steps):

$$\frac{b c - a d}{a b e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} - \frac{c^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{a^{3/2} e^{3/2}} + \frac{d^{3/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{\sqrt{b} \sqrt{e}}\right]}{b^{3/2} e^{3/2}}$$

Result (type 6, 332 leaves):

$$\frac{1}{a b e^2 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((b c - a d) (c + d x^2) + \left(2 b^2 c^2 d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) - \left(2 a^2 c d^2 x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) / \left(-4 a c \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + x^2 \left(a d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) \right) \right)$$

Problem 311: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 170 leaves, 5 steps):

$$-\frac{3 (b c - a d)}{2 a^2 e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} + \frac{b c - a d}{2 a \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(a e - \frac{c e (a + b x^2)}{c + d x^2} \right)} + \frac{3 \sqrt{c} (b c - a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{2 a^{5/2} e^{3/2}}$$

Result (type 6, 212 leaves):

$$\frac{1}{2 a^2 e^2 x^2 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(-(c + d x^2) (3 b c x^2 + a (c - 2 d x^2)) + \left(6 b c d (-b c + a d) x^4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 312: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 255 leaves, 6 steps):

$$\frac{b(b c - a d)}{a^3 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} - \frac{(b c - a d)^2 \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{4 a^2 \left(a e - \frac{c e(a+b x^2)}{c+d x^2}\right)^2} - \frac{(7 b c - 3 a d)(b c - a d) \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{8 a^3 \left(a e^2 - \frac{c e^2(a+b x^2)}{c+d x^2}\right)} - \frac{3(b c - a d)(5 b c - a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{8 a^{7/2} \sqrt{c} e^{3/2}}$$

Result (type 6, 247 leaves):

$$\frac{1}{8 a^3 e^2 x^4 (a+b x^2)} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \left((c+d x^2) (15 b^2 c x^4 + a b x^2 (5 c - 13 d x^2) - a^2 (2 c + 5 d x^2)) + \left(6 b d (5 b^2 c^2 - 6 a b c d + a^2 d^2) x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \right. \\ \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 313: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 453 leaves, 8 steps):

$$\frac{(7 b c - 8 a d) x (a+b x^2)}{5 b^3 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} + \frac{6 d x^3 (a+b x^2)}{5 b^2 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} + \frac{(b^2 c^2 - 16 a b c d + 16 a^2 d^2) x (a+b x^2)}{5 b^4 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c+d x^2)} - \frac{x^3 (c+d x^2)}{b e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} - \\ \frac{\sqrt{c} (b^2 c^2 - 16 a b c d + 16 a^2 d^2) (a+b x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 b^4 \sqrt{d} e \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c+d x^2)} - \frac{c^{3/2} (7 b c - 8 a d) (a+b x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 b^3 \sqrt{d} e \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c+d x^2)}$$

Result (type 4, 271 leaves):

$$\frac{1}{5 b^3 \sqrt{\frac{b}{a} d e^2 (a + b x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a} d x (c + d x^2) (-8 a^2 d + a b (7 c - 2 d x^2) + b^2 x^2 (2 c + d x^2)) - i c (b^2 c^2 - 16 a b c d + 16 a^2 d^2)} \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right. \\ \left. \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + i c (b^2 c^2 - 9 a b c d + 8 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 314: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(\frac{e (a + b x^2)}{c + d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 378 leaves, 7 steps):

$$\frac{4 d x (a + b x^2)}{3 b^2 e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} + \frac{d (7 b c - 8 a d) x (a + b x^2)}{3 b^3 e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} - \frac{x (c + d x^2)}{b e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} - \\ \frac{\sqrt{c} \sqrt{d} (7 b c - 8 a d) (a + b x^2) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 b^3 e \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} + \frac{c^{3/2} (3 b c - 4 a d) (a + b x^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a b^2 \sqrt{d} e \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}$$

Result (type 4, 219 leaves):

$$\frac{1}{3 a^2 \left(\frac{b}{a}\right)^{5/2} e^2 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a} x (c + d x^2) (-3 b c + 4 a d + b d x^2) + i c (-7 b c + 8 a d)} \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - \right. \\ \left. 4 i c (-b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 315: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 327 leaves, 6 steps):

$$\frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} +$$

$$\frac{\sqrt{c}\sqrt{d}(bc-2ad)(a+bx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1-\frac{bc}{ad}\right]}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Result (type 4, 203 leaves):

$$\frac{1}{a^2\left(\frac{b}{a}\right)^{3/2}e^2(a+bx^2)}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] +\right.$$

$$\left.(bc-ad\left(\sqrt{\frac{b}{a}}x(c+dx^2)-ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right]\right)\right)$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 380 leaves, 7 steps):

$$\frac{bc - ad}{abe x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc - ad)(a+bx^2)}{a^2 b e x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc - ad)x(a+bx^2)}{a^2 b e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} -$$

$$\frac{\sqrt{c} \sqrt{d} (2bc - ad)(a+bx^2) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{a^2 b e \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2} \sqrt{d} (a+bx^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{a^2 e \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Result (type 4, 223 leaves):

$$\frac{1}{a^2 \sqrt{\frac{b}{a}} e^2 x (a+bx^2)}$$

$$\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(-\sqrt{\frac{b}{a}} (c+dx^2) (ac + 2bcx^2 - adx^2) + ic(-2bc + ad)x \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] - \right.$$

$$\left. 2ic(-bc + ad)x \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}x\right], \frac{ad}{bc}\right] \right)$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 444 leaves, 8 steps):

$$\frac{bc - ad}{abe x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc - 3ad)(a+bx^2)}{3a^2 b e x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc - 7ad)(a+bx^2)}{3a^3 e x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc - 7ad)x(a+bx^2)}{3a^3 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} +$$

$$\frac{\sqrt{c} \sqrt{d} (8bc - 7ad)(a+bx^2) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{3a^3 e \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c} \sqrt{d} (4bc - 3ad)(a+bx^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}\right]}{3a^3 e \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Result (type 4, 266 leaves):

$$\frac{1}{3 a^3 \sqrt{\frac{b}{a}} e^2 x^3 (a + b x^2)}$$

$$\sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(-\sqrt{\frac{b}{a}} (c + d x^2) (-8 b^2 c x^4 + a^2 (c + 4 d x^2) + a b (-4 c x^2 + 7 d x^4)) - i b c (-8 b c + 7 a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right.$$

$$\left. \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - i (8 b^2 c^2 - 11 a b c d + 3 a^2 d^2) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 321: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c + d x^2}}}{x} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{a}}\right] - \frac{\sqrt{b + a c} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{b + a c}}\right]}{\sqrt{c}}$$

Result (type 3, 210 leaves):

$$\left(\sqrt{c + d x^2} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left((b + a c) \text{Log}[x^2] + 2 \sqrt{a} \sqrt{c} \sqrt{b + a c} \text{Log}\left[a \sqrt{c + d x^2} + \sqrt{a} \sqrt{b + a c + a d x^2}\right] - \right. \right.$$

$$\left. \left. (b + a c) \text{Log}\left[2 a c (c + d x^2) + b (2 c + d x^2) + 2 \sqrt{c} \sqrt{b + a c} \sqrt{c + d x^2} \sqrt{b + a c + a d x^2}\right] \right) \right) / \left(2 \sqrt{c} \sqrt{b + a c} \sqrt{b + a (c + d x^2)} \right)$$

Problem 322: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c + d x^2}}}{x^3} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} + \frac{bd \operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2c^{3/2}\sqrt{b+ac}}$$

Result (type 3, 212 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(-2\sqrt{c(b+ac)}(c+dx^2)(b+ac+adx^2) - 2bdx^2\sqrt{(c+dx^2)(b+a(c+dx^2))} \operatorname{Log}[x] + bdx^2\sqrt{(c+dx^2)(b+ac+adx^2)} \right. \right. \\ \left. \left. \operatorname{Log}\left[2ac(c+dx^2) + b(2c+dx^2) + 2\sqrt{c(b+ac)}\sqrt{(c+dx^2)(b+ac+adx^2)}\right] \right) \right) / \left(4c\sqrt{c(b+ac)}x^2(b+a(c+dx^2)) \right)$$

Problem 325: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$-\frac{(2b^2+7abc-3a^2c^2)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2} + \frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} + \\ \frac{\sqrt{c}(2b^2+7abc-3a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(b-3ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 293 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x (c+dx^2) (b^2 - 2ab(c-2dx^2) - 3a^2(c^2 - d^2x^4)) + \right. \right. \\ \left. \left. i c (2b^2 + 7abc - 3a^2c^2) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] - \right. \right. \\ \left. \left. i bc (b+9ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] \right) \right) / \left(15ad^2 \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 326: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\frac{(b-ac)x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad} + \frac{x(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \\ \frac{\sqrt{c}(b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}} \right], \frac{b}{b+ac} \right]}{3ad^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}} \right], \frac{b}{b+ac} \right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 250 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x (c+dx^2) (b+ac+adx^2) + i c (-b+ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] + \right. \right. \\ \left. \left. 2 i bc \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] \right) \right) / \left(3d \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^2} dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\frac{dx \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}$$

$$\frac{\sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] + a\sqrt{c} \sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{c} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} + (b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 141 leaves):

$$\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(-\frac{1}{x} - \frac{dx}{c} - \frac{i a d \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1 + \frac{b}{ac}\right]}{\sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2))} \right)$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} dx$$

Optimal (type 4, 362 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} + \\
& \frac{(2b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] - ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} - 3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 314 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{ad}{b+ac}} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} (c+dx^2) (b^2(c-2dx^2) + a^2c(c^2-d^2x^4) + 2ab(c^2-cdx^2-d^2x^4)) - \right. \right. \right. \\
& \quad \left. \left. \left. iac(2b+ac)d^2x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. iabc d^2 x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right] \right) \right) \right) / (3ac^2dx^3(b+a(c+dx^2)))
\end{aligned}$$

Problem 330: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^6} dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\frac{(8b^2 + 13abc + 3a^2c^2)d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+a)^2} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} +$$

$$\frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+a)^3} - \frac{(8b^2 + 13abc + 3a^2c^2)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+a)^2x} -$$

$$\frac{(8b^2 + 13abc + 3a^2c^2)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{5/2}(b+a)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+a)(c+dx^2)}}} + \frac{a(4b+3ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{3/2}(b+a)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+a)(c+dx^2)}}}$$

Result (type 4, 402 leaves):

$$-\frac{1}{15c^3(b+a)^2\sqrt{\frac{ad}{b+ac}}x^5(b+a(c+dx^2))}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}(c+dx^2)\right.$$

$$\left.(b^3(3c^2-4cdx^2+8d^2x^4)+3a^3c^2(c^3+d^3x^6)+ab^2(9c^3-8c^2dx^2+17cd^2x^4+8d^3x^6)+a^2bc(9c^3-4c^2dx^2+9cd^2x^4+13d^3x^6)\right) +$$

$$i ac(8b^2+13abc+3a^2c^2)d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right] -$$

$$2iabc(2b+3ac)d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right]$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$\frac{(b^2 - 14abc + a^2c^2) x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2} + \frac{(7b-ac) x (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2} + \frac{6ax^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{x^3 (b+ac+adx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}$$

$$\frac{\sqrt{c} (b^2 - 14abc + a^2c^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5ad^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2} (7b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5d^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 308 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x \left(-a^2 (c-dx^2) (c+dx^2)^2 + b^2 (7c+2dx^2) + 3ab (2c^2 + 3cdx^2 + d^2x^4) \right) - \right. \right.$$

$$\left. \left. i c (b^2 - 14abc + a^2c^2) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] + \right. \right.$$

$$\left. \left. 8 i bc (b-ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x \right], 1 + \frac{b}{ac} \right] \right) \right) / \left(5d^2 \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(a + \frac{b}{c+dx^2} \right)^{3/2} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\frac{(7b-ac) x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} + \frac{4ax (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{x (b+ac+adx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}$$

$$\frac{\sqrt{c} (7b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c} (3b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 270 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x (-3b^2 - 2ab(c+dx^2) + a^2(c+dx^2)^2) + \right. \right. \\ \left. \left. iac(-7b+ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] + \right. \right. \\ \left. \left. ib(-3b+5ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] \right) \right) / \left(3d \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 340: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + \frac{b}{c+dx^2} \right)^{3/2} dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\frac{bx \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(b-ac)x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \\ \frac{(b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] + a\sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{c} \sqrt{d} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{d} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 243 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(b \sqrt{\frac{ad}{b+ac}} x (b+a(c+dx^2)) - iac(-b+ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] - \right. \right. \\ \left. \left. 2iac \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] \right) \right) / \left(c \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 312 leaves, 8 steps):

$$\frac{b \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{(2b+ac) dx \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2} - \frac{(2b+ac)(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2 x} -$$

$$\frac{(2b+ac) \sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] + a \sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{c^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} + \sqrt{c} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 278 leaves):

$$- \left(\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} \left(2ab(c+dx^2)^2 + a^2c(c+dx^2)^2 + b^2(c+2dx^2) \right) + \right. \right. \right.$$

$$\left. \left. \left. i a c (2b+ac) dx \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1 + \frac{b}{ac}\right] - \right. \right. \right.$$

$$\left. \left. \left. i a b c d x \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1 + \frac{b}{ac}\right] \right) \right) \right) / \left(c^2 \sqrt{\frac{ad}{b+ac}} x (b+a(c+dx^2)) \right)$$

Problem 342: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 388 leaves, 9 steps):

$$\frac{b \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(8b+ac)d^2x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3} - \frac{(4b+ac)(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} +$$

$$\frac{(8b+ac)d^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] - a(4b+ac)d^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} - 3c^{3/2}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 329 leaves):

$$-\frac{1}{3c^3 \sqrt{\frac{ad}{b+ac}} x^3 (b+a(c+dx^2))}$$

$$\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} (a^2c(c-dx^2)(c+dx^2)^2 + b^2(c^2 - 4cdx^2 - 8d^2x^4) + ab(2c^3 - 3c^2dx^2 - 13cd^2x^4 - 8d^3x^6)) - \right.$$

$$\left. iac(8b+ac)d^2x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1 + \frac{b}{ac}\right] + \right.$$

$$\left. 4iabc d^2x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1 + \frac{b}{ac}\right] \right)$$

Problem 343: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx$$

Optimal (type 4, 494 leaves, 10 steps):

$$\begin{aligned}
& \frac{b \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} + \frac{(16b^2 + 16abc + a^2c^2) d^3 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)} - \frac{(6b+ac)(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \\
& \frac{(8b+ac)d(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3} - \frac{(16b^2 + 16abc + a^2c^2) d^2 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x} - \\
& \frac{(16b^2 + 16abc + a^2c^2) d^{5/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5c^{7/2}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a(8b+ac) d^{5/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5c^{5/2}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 430 leaves):

$$\begin{aligned}
& - \frac{1}{5ac^4dx^5(b+a(c+dx^2))} \sqrt{\frac{ad}{b+ac}} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \\
& \left(\sqrt{\frac{ad}{b+ac}} (b^3(c^3 - 2c^2dx^2 + 8cd^2x^4 + 16d^3x^6) + a^3c^2(c^4 + c^3dx^2 + cd^3x^6 + d^4x^8) + a^2bc(3c^4 + 5c^2d^2x^4 + 24cd^3x^6 + 16d^4x^8) + \right. \\
& \quad \left. ab^2(3c^4 - 3c^3dx^2 + 13c^2d^2x^4 + 40cd^3x^6 + 16d^4x^8)) + \right. \\
& \quad \left. iac(16b^2 + 16abc + a^2c^2) d^3x^5 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1 + \frac{b}{ac}\right] - \right. \\
& \quad \left. iabc(8b+7ac) d^3x^5 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1 + \frac{b}{ac}\right] \right)
\end{aligned}$$

Problem 347: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{\sqrt{c} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{\sqrt{b+ac}}$$

Result (type 3, 210 leaves):

$$\left(\sqrt{b+ac+adx^2} \left(\sqrt{a} \sqrt{c} \text{Log}[x^2] + 2\sqrt{b+ac} \text{Log}\left[a\sqrt{c+dx^2} + \sqrt{a} \sqrt{b+ac+adx^2}\right] - \sqrt{a} \sqrt{c} \text{Log}\left[2ac(c+dx^2) + b(2c+dx^2) + 2\sqrt{c} \sqrt{b+ac} \sqrt{c+dx^2} \sqrt{b+ac+adx^2}\right]\right)\right) / \left(2\sqrt{a} \sqrt{b+ac} \sqrt{c+dx^2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 443 leaves, 8 steps):

$$\begin{aligned} & -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(8b^2+13abc+3a^2c^2)x(b+ac+adx^2)}{15a^3d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ & \frac{\sqrt{c}(8b^2+13abc+3a^2c^2)(b+ac+adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^3d^{5/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(4b+3ac)(b+ac+adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^2d^{5/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \end{aligned}$$

Result (type 4, 297 leaves):

$$\begin{aligned} & -\left(\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x(c+dx^2) (4b^2+ab(7c+dx^2)+3a^2(c^2-d^2x^4)) + \right.\right.\right. \\ & \quad \left.\left.\left. i c (8b^2+13abc+3a^2c^2) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] - \right.\right.\right. \\ & \quad \left.\left.\left. 2 i b c (2b+3ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right]\right)\right) / \left(15a^2d^2 \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2))\right) \end{aligned}$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 354 leaves, 7 steps):

$$\frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(2b+ac)x(b+ac+adx^2)}{3a^2d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} +$$

$$\frac{\sqrt{c}(2b+ac)(b+ac+adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] - c^{3/2}(b+ac+adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3a^2d^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} - 3ad^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 253 leaves):

$$\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} x(c+dx^2)(b+ac+adx^2) + ic(2b+ac) \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] - \right. \right.$$

$$\left. \left. ic \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] \right) \right) / \left(3ad \sqrt{\frac{ad}{b+ac}} (b+a(c+dx^2)) \right)$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 431 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b + a c + a d x^2}{3 (b + a c) x^3 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \frac{(b - a c) d (b + a c + a d x^2)}{3 c (b + a c)^2 x \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{(b - a c) d^2 x (b + a c + a d x^2)}{3 c (b + a c)^2 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \\
& \frac{(b - a c) d^{3/2} (b + a c + a d x^2) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 \sqrt{c} (b + a c)^2 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} - \frac{a \sqrt{c} d^{3/2} (b + a c + a d x^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 (b + a c)^2 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 314 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(- \sqrt{\frac{a d}{b + a c}} (c + d x^2) (b^2 (c + d x^2) + a^2 c (c^2 - d^2 x^4) + a b (2 c^2 + c d x^2 + d^2 x^4)) + \right. \right. \\
& \quad \left. \left. i a c (-b + a c) d^2 x^3 \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}\right] + \right. \right. \\
& \quad \left. \left. 2 i a b c d^2 x^3 \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}\right] \right) \right) / \left(3 c (b + a c)^2 \sqrt{\frac{a d}{b + a c}} x^3 (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 358: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a + \frac{b}{c + d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$- \frac{b}{a (b + a c) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{c^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{b + a c}}\right]}{(b + a c)^{3/2}}$$

Result (type 3, 306 leaves):

$$\frac{1}{2 a^{3/2} (b+a c)^{3/2} (b+a (c+d x^2))} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(a^{3/2} c^{3/2} \sqrt{c+d x^2} \sqrt{b+a (c+d x^2)} \operatorname{Log}[x^2] + \right. \\ \left. (b+a c)^{3/2} \sqrt{c+d x^2} \sqrt{b+a (c+d x^2)} \operatorname{Log}[b+2 a (c+d x^2)+2 \sqrt{a} \sqrt{c+d x^2} \sqrt{b+a (c+d x^2)}] - \sqrt{a} \right. \\ \left. \left(2 b \sqrt{b+a c} (c+d x^2) + a c^{3/2} \sqrt{c+d x^2} \sqrt{b+a (c+d x^2)} \operatorname{Log}[2 a c (c+d x^2)+b (2 c+d x^2)+2 \sqrt{c} \sqrt{b+a c} \sqrt{c+d x^2} \sqrt{b+a (c+d x^2)}] \right) \right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 482 leaves, 9 steps):

$$\frac{-\frac{x^3 (c+d x^2)}{a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} - \frac{(8 b+a c) x (b+a c+a d x^2)}{5 a^3 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{6 x^3 (b+a c+a d x^2)}{5 a^2 d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(16 b^2+16 a b c+a^2 c^2) x (b+a c+a d x^2)}{5 a^4 d^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}}{\sqrt{c} (16 b^2+16 a b c+a^2 c^2) (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right] + \frac{c^{3/2} (8 b+a c) (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a^4 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}} + \frac{5 a^3 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}{5 a^3 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}}$$

Result (type 4, 296 leaves):

$$-\left(\left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(\sqrt{\frac{a d}{b+a c}} x (c+d x^2) (8 b^2+a b (9 c+2 d x^2)+a^2 (c^2-d^2 x^4)) + \right. \right. \right. \\ \left. \left. \left. i c (16 b^2+16 a b c+a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] - \right. \right. \right. \\ \left. \left. \left. i b c (8 b+7 a c) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] \right) \right) / \left(5 a^3 d^2 \sqrt{\frac{a d}{b+a c}} (b+a (c+d x^2)) \right) \right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 409 leaves, 8 steps):

$$\begin{aligned} & -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{3a^3d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \\ & \frac{\sqrt{c}(8b+ac)(b+ac+adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] - c^{3/2}(4b+ac)(b+ac+adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3a^3d^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} - 3a^2(b+ac)d^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \end{aligned}$$

Result (type 4, 255 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \right. \\ & \left. \left(\sqrt{\frac{ad}{b+ac}} x(c+dx^2)(4b+ac+adx^2) + i(8b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right] - \right. \right. \\ & \left. \left. 4ibc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}}x\right], 1+\frac{b}{ac}\right]\right) \right) / \left(3a^2d\sqrt{\frac{ad}{b+ac}}(b+a(c+dx^2)) \right) \end{aligned}$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$\begin{aligned}
& - \frac{b x}{a (b+a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(2 b+a c) x (b+a c+a d x^2)}{a^2 (b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \\
& \frac{\sqrt{c} (2 b+a c) (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a^2 (b+a c) \sqrt{d} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{c^{3/2} (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a (b+a c) \sqrt{d} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 241 leaves):

$$\begin{aligned}
& - \frac{1}{a^2 d (b+a (c+d x^2))} \\
& \sqrt{\frac{a d}{b+a c}} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(b \sqrt{\frac{a d}{b+a c}} x (c+d x^2) + i c (2 b+a c) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] - \right. \\
& \left. i b c \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] \right)
\end{aligned}$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 410 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b}{a (b+a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(b-a c) (b+a c+a d x^2)}{a (b+a c)^2 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(b-a c) d x (b+a c+a d x^2)}{a (b+a c)^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \\
& \frac{\sqrt{c} (b-a c) \sqrt{d} (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a (b+a c)^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{c^{3/2} \sqrt{d} (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c)^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 268 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} (c+dx^2) (b(c-dx^2) + ac(c+dx^2)) + \right. \right. \right. \\
& \quad \left. \left. \left. i c (-b+ac) dx \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 i b c d x \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] \right) \right) / \left((b+ac)^2 \sqrt{\frac{ad}{b+ac}} x (b+a(c+dx^2)) \right) \right)
\end{aligned}$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 490 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b}{a(b+ac)x^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)(b+ac+adx^2)}{3a(b+ac)^2 x^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(7b-ac)d^2 x(b+ac+adx^2)}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \\
& \frac{\sqrt{c}(7b-ac)d^{3/2}(b+ac+adx^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 319 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} (c+dx^2) (b^2(c+4dx^2) + a^2c(c^2-d^2x^4) + ab(2c^2+4c dx^2+7d^2x^4)) - \right. \right. \right. \\
& \quad \left. \left. \left. i a c (-7b+ac) d^2 x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] + i b (3b-5ac) d^2 x^3 \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}\right] \right) \right) / \left(3(b+ac)^3 \sqrt{\frac{ad}{b+ac}} x^3 (b+a(c+dx^2)) \right) \right)
\end{aligned}$$

Problem 366: Unable to integrate problem.

$$\int \frac{\sqrt{a x^{23}}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 75 leaves, 6 steps):

$$-\frac{3 \sqrt{a x^{23}} \sqrt{1+x^5}}{20 x^9} + \frac{\sqrt{a x^{23}} \sqrt{1+x^5}}{10 x^4} + \frac{3 \sqrt{a x^{23}} \operatorname{ArcSinh}\left[x^{5/2}\right]}{20 x^{23/2}}$$

Result (type 8, 21 leaves):

$$\int \frac{\sqrt{a x^{23}}}{\sqrt{1+x^5}} dx$$

Problem 367: Unable to integrate problem.

$$\int \frac{\sqrt{a x^{13}}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 50 leaves, 5 steps):

$$\frac{\sqrt{a x^{13}} \sqrt{1+x^5}}{5 x^4} - \frac{\sqrt{a x^{13}} \operatorname{ArcSinh}\left[x^{5/2}\right]}{5 x^{13/2}}$$

Result (type 8, 21 leaves):

$$\int \frac{\sqrt{a x^{13}}}{\sqrt{1+x^5}} dx$$

Problem 368: Unable to integrate problem.

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{2 \sqrt{a x^3} \operatorname{ArcSinh}\left[x^{5/2}\right]}{5 x^{3/2}}$$

Result (type 8, 21 leaves):

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^5}} dx$$

Problem 374: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{a x^6}}{x(1-x^4)} \right) dx$$

Optimal (type 3, 49 leaves, 8 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{a x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{a x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 35 leaves):

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{a x^6}}{x(1-x^4)} \right) dx$$

Problem 375: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{a x^6}}{x-x^5} \right) dx$$

Optimal (type 3, 49 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{a x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{a x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 32 leaves):

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{a x^6}}{x-x^5} \right) dx$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$\frac{2 \sqrt{a x^3} \sqrt{1+x^2}}{3 x} - \frac{\sqrt{a x^3} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[\sqrt{x}], \frac{1}{2}\right]}{3 x^{3/2} \sqrt{1+x^2}}$$

Result (type 4, 77 leaves):

$$\frac{2 \sqrt{a x^3} \sqrt{1+x^2} \left(\sqrt{1 + \frac{1}{x^2}} x^{3/2} - (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right] \right)}{3 \sqrt{1 + \frac{1}{x^2}} x^{5/2}}$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{2 \sqrt{a x} \sqrt{1+x^2}}{1+x} - \frac{2 \sqrt{a} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{a x}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} + \frac{\sqrt{a} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{a x}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 58 leaves):

$$\frac{2 (-1)^{3/4} \sqrt{a x} \left(-\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right)}{\sqrt{x}}$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{\sqrt{\frac{a}{x}} \sqrt{x} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[\sqrt{x}], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 57 leaves):

$$\frac{2 (-1)^{1/4} \sqrt{\frac{a}{x}} \sqrt{1+x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right]}{\sqrt{1 + \frac{1}{x^2}} \sqrt{x}}$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 159 leaves, 6 steps):

$$-2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^2} + \frac{2 \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^2}}{1+x} -$$

$$\frac{2 \sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}[\sqrt{x}], \frac{1}{2}\right]}{\sqrt{1+x^2}} + \frac{\sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[\sqrt{x}], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 74 leaves):

$$2 \sqrt{\frac{a}{x^3}} x \left(-\sqrt{1+x^2} + (-1)^{3/4} \sqrt{x} \left(-\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right) \right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 292 leaves, 5 steps):

$$\frac{(1 + \sqrt{3}) \sqrt{ax^3} \sqrt{1+x^3}}{x(1 + (1 + \sqrt{3})x)} - \frac{3^{1/4} \sqrt{ax^3} (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right], \frac{1}{4}(2 + \sqrt{3})\right]}{x \sqrt{\frac{x(1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3}}$$

$$\frac{(1 - \sqrt{3}) \sqrt{ax^3} (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right], \frac{1}{4}(2 + \sqrt{3})\right]}{2 \times 3^{1/4} x \sqrt{\frac{x(1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 174 leaves):

$$\frac{1}{\sqrt{ax^3} \sqrt{1+x^3}} a x \left(1 + x^3 + \frac{1}{\sqrt{6}} (1 - (-1)^{2/3}) x^2 \sqrt{\frac{-(-1)^{1/3} + x}{(1 + (-1)^{1/3})x}} \sqrt{\frac{(1+x)(-1 + i\sqrt{3} + 2x)}{x^2}} \right. \\ \left. \left((1 + (-1)^{1/3}) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1 + (-1)^{2/3})x}}\right], 1 + (-1)^{2/3}\right] - \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1 + (-1)^{2/3})x}}\right], 1 + (-1)^{2/3}\right] \right) \right)$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\frac{2\sqrt{ax^2} \sqrt{1+x^3}}{x(1 + \sqrt{3} + x)} - \frac{3^{1/4} \sqrt{2 - \sqrt{3}} \sqrt{ax^2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7 - 4\sqrt{3}\right]}{x \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} +$$

$$\frac{2\sqrt{2} \sqrt{ax^2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7 - 4\sqrt{3}\right]}{3^{1/4} x \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 134 leaves):

$$-\frac{1}{3^{1/4} \sqrt{a x^2} \sqrt{1+x^3}} 2 a x \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x \right) \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2}}$$

$$\left(\sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{5/6} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 116 leaves, 3 steps):

$$\frac{\sqrt{\frac{a}{x}} x (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcCos} \left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x} \right], \frac{1}{4} (2+\sqrt{3}) \right]}{3^{1/4} \sqrt{\frac{x(1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 106 leaves):

$$-\frac{1}{3^{1/4} \sqrt{1+x^3}} 2 (-1)^{1/6} \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + \frac{1}{x} \right) \sqrt{1 + \frac{(-1)^{2/3}}{x^2} + \frac{(-1)^{1/3}}{x}}} \sqrt{\frac{a}{x}} x^2 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} \left(1 + \frac{1}{x} \right)}}{3^{1/4}} \right], (-1)^{1/3} \right]$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$\begin{aligned}
& -2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^3} + \frac{2(1+\sqrt{3}) \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^3}}{1+(1+\sqrt{3})x} - \frac{2 \times 3^{1/4} \sqrt{\frac{a}{x^3}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{\sqrt{\frac{x(1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3}} \\
& \frac{(1-\sqrt{3}) \sqrt{\frac{a}{x^3}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{3^{1/4} \sqrt{\frac{x(1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 165 leaves):

$$\begin{aligned}
& -\frac{1}{\sqrt{\frac{a}{x^3}} \sqrt{1+x^3}} \sqrt{\frac{2}{3}} (-1+(-1)^{2/3}) a \sqrt{\frac{-(-1)^{1/3}+x}{(1+(-1)^{1/3})x}} \sqrt{\frac{(1+x)(-1+i\sqrt{3}+2x)}{x^2}} \\
& \left((1+(-1)^{1/3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1+(-1)^{2/3})x}}\right], 1+(-1)^{2/3}\right] - \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1+(-1)^{2/3})x}}\right], 1+(-1)^{2/3}\right] \right)
\end{aligned}$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 281 leaves, 5 steps):

$$\begin{aligned}
& -\sqrt{\frac{a}{x^4}} x \sqrt{1+x^3} + \frac{\sqrt{\frac{a}{x^4}} x^2 \sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{2 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \frac{\sqrt{2} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 146 leaves):

$$\frac{1}{3\sqrt{1+x^3}} \sqrt{\frac{a}{x^4}} x \left(-3(1+x^3) - 3^{3/4} x \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x \right)} \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \right. \\ \left. \left(\sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{5/6} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex} \sqrt{e+fx}} dx$$

Optimal (type 4, 114 leaves, 2 steps):

$$\frac{2\sqrt{-e^2+df} \sqrt{ax} \sqrt{\frac{e(e+fx)}{e^2-df}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{f} \sqrt{d+ex}}{\sqrt{-e^2+df}} \right], 1 - \frac{e^2}{df} \right]}{e\sqrt{f} \sqrt{-\frac{ex}{d}} \sqrt{e+fx}}$$

Result (type 4, 106 leaves):

$$\frac{2i e \sqrt{ax} \sqrt{1 + \frac{fx}{e}} \left(\operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{ex}{d}} \right], \frac{df}{e^2} \right] - \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{ex}{d}} \right], \frac{df}{e^2} \right] \right)}{f \sqrt{\frac{ex}{d+ex}} \sqrt{d+ex} \sqrt{e+fx}}$$

Problem 423: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx$$

Optimal (type 3, 32 leaves, 6 steps):

$$2\sqrt{1-x^2} - 2 \operatorname{ArcTanh} \left[\sqrt{1-x^2} \right] + 2 \operatorname{Log} [x]$$

Result (type 3, 84 leaves):

$$2 \left(\sqrt{1-x^2} + \operatorname{Log} [-x] + \operatorname{Log} [1 - \sqrt{1+x}] - \operatorname{Log} [2 + \sqrt{1-x} - \sqrt{1+x}] - \operatorname{Log} [1 + \sqrt{1+x}] + \operatorname{Log} [2 + \sqrt{1-x} + \sqrt{1+x}] \right)$$

Problem 425: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx$$

Optimal (type 3, 34 leaves, 6 steps):

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} + \text{ArcTanh}[\sqrt{1-x^2}]$$

Result (type 3, 88 leaves):

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} - \text{Log}[1 - \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] + \text{Log}[1 + \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 448: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x} dx$$

Optimal (type 3, 32 leaves, 7 steps):

$$-2\sqrt{1-x^2} + 2\text{ArcTanh}[\sqrt{1-x^2}] - 2\text{Log}[x]$$

Result (type 3, 84 leaves):

$$-2 \left(\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}] \right)$$

Problem 450: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x^3} dx$$

Optimal (type 3, 33 leaves, 7 steps):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} - \text{ArcTanh}[\sqrt{1-x^2}]$$

Result (type 3, 85 leaves):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 451: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx$$

Optimal (type 3, 28 leaves, 15 steps):

$$\sqrt{1-x^2} - \text{ArcTanh}[\sqrt{1-x^2}] + \text{Log}[x]$$

Result (type 3, 82 leaves):

$$\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 453: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 5, 121 leaves, 4 steps):

$$\frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1+n)} + \frac{a f^2 \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{d+e x+f \sqrt{a + \frac{e^2 x^2}{f^2}}}{d}\right]}{2 d^2 e (1+n)}$$

Result (type 8, 27 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 460: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 225 leaves, 6 steps):

$$\frac{2 a d f^2 \sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{e} - \frac{a d^2 f^2 \sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{2 e \left(e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)} +$$

$$\frac{a f^2 \left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{3/2}}{3 e} + \frac{\left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{7/2}}{7 e} - \frac{5 a d^{3/2} f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{\sqrt{d}} \right]}{2 e}$$

Result (type 8, 29 leaves):

$$\int \left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Problem 461: Unable to integrate problem.

$$\int \left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal (type 3, 183 leaves, 6 steps):

$$\frac{a f^2 \sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{e} - \frac{a d f^2 \sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{2 e \left(e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)} + \frac{\left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{5/2}}{5 e} - \frac{3 a \sqrt{d} f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d+e x+f} \sqrt{a+\frac{e^2 x^2}{f^2}}}{\sqrt{d}} \right]}{2 e}$$

Result (type 8, 29 leaves):

$$\int \left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 464: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Optimal (type 3, 158 leaves, 5 steps):

$$-\frac{1 + \frac{a f^2}{d^2}}{e \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)} + \frac{3 a f^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 d^{5/2} e}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Problem 465: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Optimal (type 3, 199 leaves, 6 steps):

$$-\frac{1 + \frac{a f^2}{d^2}}{3 e \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} - \frac{2 a f^2}{d^3 e \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^3 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)} + \frac{5 a f^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 d^{7/2} e}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Problem 472: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 5, 166 leaves, 4 steps):

$$\frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{2 e (1+n)} + \frac{1}{2 e (2 d e - b f^2)^2 (1+n)}$$

$$f^2 (4 a e^2 - b^2 f^2) \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{2 e \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)}{2 d e - b f^2}\right]$$

Result (type 8, 30 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 478: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^3} dx$$

Optimal (type 3, 330 leaves, 3 steps):

$$\begin{aligned}
& - \frac{d^2 e - b d f^2 + a e f^2}{(2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^2} - \frac{2 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)} - \frac{2 e f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \\
& \frac{6 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log} \left[d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right]}{(2 d e - b f^2)^4} - \frac{6 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log} \left[b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right]}{(2 d e - b f^2)^4}
\end{aligned}$$

Result (type 3, 665 leaves):

$$\begin{aligned}
& \frac{4 e^3 x}{(2 d e - b f^2)^3} - \frac{2 (d^2 e - b d f^2 + a e f^2)^3}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))^2} - \frac{3 (4 a^2 e^3 f^4 + b^2 d f^4 (-d e + b f^2) + a e f^2 (4 d^2 e^2 - 4 b d e f^2 - b^2 f^4))}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))} - \\
& \left(2 f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} (b^3 f^6 x + b e f^2 (-3 d^3 - a d f^2 + d^2 e x - 9 a e f^2 x + 8 d e^2 x^2) + b^2 (a f^6 - e f^4 x (d + 2 e x)) - \right. \\
& \left. 2 e^2 (3 a^2 f^4 + d^2 e x (3 d + 4 e x) - a d f^2 (5 d + 9 e x))) \right) / \left((-2 d e + b f^2)^3 (d^2 + 2 d e x - f^2 (a + b x))^2 \right) - \\
& \frac{3 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log} [d^2 + 2 d e x - f^2 (a + b x)]}{(-2 d e + b f^2)^4} + \frac{3 (4 a e^3 f^2 - b^2 e f^4) \operatorname{Log} [d^2 + 2 d e x - f^2 (a + b x)]}{(-2 d e + b f^2)^4} - \\
& \frac{3 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log} \left[b f^2 + 2 e \left(e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) \right]}{(-2 d e + b f^2)^4} + \frac{1}{(-2 d e + b f^2)^4} 3 e f^2 (4 a e^2 - b^2 f^2) \\
& \left. \operatorname{Log} \left[b^2 f^4 x + 2 d^2 e \left(e x - f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) - 2 a e f^2 \left(2 d + e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) + b f^2 \left(d^2 + a f^2 - 2 d e x + 2 d f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) \right] \right)
\end{aligned}$$

Problem 479: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 370 leaves, 6 steps):

$$\frac{f^2 (2de - b^2 f^2) (4ae^2 - b^2 f^2) \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{4e^4} + \frac{f^2 (4ae^2 - b^2 f^2) \left(d+ex+f \sqrt{a+bx+\frac{e^2 x^2}{f^2}}\right)^{3/2}}{12e^3} + \frac{\left(d+ex+f \sqrt{a+bx+\frac{e^2 x^2}{f^2}}\right)^{7/2}}{7e}$$

$$\frac{f^2 (2de - b^2 f^2)^2 (4ae^2 - b^2 f^2) \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{16e^4 \left(bf^2 + 2e \left(ex+f \sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{5f^2 (2de - b^2 f^2)^{3/2} (4ae^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{e} \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{\sqrt{2de-bf^2}}\right]}{16\sqrt{2}e^{9/2}}$$

Result (type 8, 32 leaves):

$$\int \left(d+ex+f \sqrt{a+bx+\frac{e^2 x^2}{f^2}}\right)^{5/2} dx$$

Problem 480: Unable to integrate problem.

$$\int \left(d+ex+f \sqrt{a+bx+\frac{e^2 x^2}{f^2}}\right)^{3/2} dx$$

Optimal (type 3, 302 leaves, 6 steps):

$$\frac{f^2 (4ae^2 - b^2 f^2) \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{4e^3} + \frac{\left(d+ex+f \sqrt{a+bx+\frac{e^2 x^2}{f^2}}\right)^{5/2}}{5e}$$

$$\frac{f^2 (2de - b^2 f^2) (4ae^2 - b^2 f^2) \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{8e^3 \left(bf^2 + 2e \left(ex+f \sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{3f^2 \sqrt{2de - b^2 f^2} (4ae^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{e} \sqrt{d+ex+f} \sqrt{a+bx+\frac{e^2 x^2}{f^2}}}{\sqrt{2de-bf^2}}\right]}{8\sqrt{2}e^{7/2}}$$

Result (type 8, 32 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 483: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Optimal (type 3, 269 leaves, 5 steps):

$$\frac{4 (d^2 e - b d f^2 + a e f^2)}{(2 d e - b f^2)^2 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} - \frac{f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{(2 d e - b f^2)^2 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \frac{3 f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}} \right]}{\sqrt{2} \sqrt{e} (2 d e - b f^2)^{5/2}}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Problem 484: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx$$

Optimal (type 3, 335 leaves, 6 steps):

$$\begin{aligned}
& - \frac{4 (d^2 e - b d f^2 + a e f^2)}{3 (2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} - \frac{4 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} \\
& \frac{2 e f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \frac{5 \sqrt{2} \sqrt{e} f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}} \right]}{(2 d e - b f^2)^{7/2}}
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx$$

Problem 485: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 \left(x + \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 164 leaves, 3 steps):

$$-\frac{a^5 \left(x + \sqrt{a + x^2} \right)^{-5+n}}{32 (5-n)} - \frac{5 a^4 \left(x + \sqrt{a + x^2} \right)^{-3+n}}{32 (3-n)} - \frac{5 a^3 \left(x + \sqrt{a + x^2} \right)^{-1+n}}{16 (1-n)} + \frac{5 a^2 \left(x + \sqrt{a + x^2} \right)^{1+n}}{16 (1+n)} + \frac{5 a \left(x + \sqrt{a + x^2} \right)^{3+n}}{32 (3+n)} + \frac{\left(x + \sqrt{a + x^2} \right)^{5+n}}{32 (5+n)}$$

Result (type 3, 338 leaves):

$$\frac{1}{2} \left(x + \sqrt{a+x^2} \right)^n \left(-\frac{2a^2 (x - n\sqrt{a+x^2})}{-1+n^2} + \right.$$

$$\frac{1}{16} \left(\frac{a^5}{(-5+n)(x+\sqrt{a+x^2})^5} - \frac{3a^4}{(-3+n)(x+\sqrt{a+x^2})^3} + \frac{2a^3}{(-1+n)(x+\sqrt{a+x^2})} + \frac{2a^2(x+\sqrt{a+x^2})}{1+n} - \frac{3a(x+\sqrt{a+x^2})^3}{3+n} + \frac{(x+\sqrt{a+x^2})^5}{5+n} \right) +$$

$$\left(4a\sqrt{a+x^2} \left(2a^3n + a^2(-3+n)nx \left((-3+n)x - 2\sqrt{a+x^2} \right) + 4(3-n-3n^2+n^3)x^5 \left(x + \sqrt{a+x^2} \right) + \right. \right.$$

$$\left. \left. a(3-4n+n^2)x^3 \left((3+5n)x + (1+3n)\sqrt{a+x^2} \right) \right) \right) / \left((-3+n)(-1+n)(1+n)(3+n) \left(x + \sqrt{a+x^2} \right)^2 \left(a+x \left(x + \sqrt{a+x^2} \right) \right) \right)$$

Problem 488: Unable to integrate problem.

$$\int \frac{(x + \sqrt{a+x^2})^n}{a+x^2} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{2(x + \sqrt{a+x^2})^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{(x+\sqrt{a+x^2})^2}{a}\right]}{a(1+n)}$$

Result (type 8, 23 leaves):

$$\int \frac{(x + \sqrt{a+x^2})^n}{a+x^2} dx$$

Problem 489: Unable to integrate problem.

$$\int \frac{(x + \sqrt{a+x^2})^n}{(a+x^2)^2} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{8(x + \sqrt{a+x^2})^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{(x+\sqrt{a+x^2})^2}{a}\right]}{a^3(3+n)}$$

Result (type 8, 23 leaves):

$$\int \frac{(x + \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Problem 490: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 (x - \sqrt{a + x^2})^n dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{a^5 (x - \sqrt{a + x^2})^{-5+n}}{32 (5 - n)} - \frac{5 a^4 (x - \sqrt{a + x^2})^{-3+n}}{32 (3 - n)} - \frac{5 a^3 (x - \sqrt{a + x^2})^{-1+n}}{16 (1 - n)} + \frac{5 a^2 (x - \sqrt{a + x^2})^{1+n}}{16 (1 + n)} + \frac{5 a (x - \sqrt{a + x^2})^{3+n}}{32 (3 + n)} + \frac{(x - \sqrt{a + x^2})^{5+n}}{32 (5 + n)}$$

Result (type 3, 361 leaves):

$$\frac{1}{2} (x - \sqrt{a + x^2})^n \left(-\frac{2 a^2 (x + n \sqrt{a + x^2})}{-1 + n^2} + \frac{1}{16} \left(\frac{a^5}{(-5 + n) (x - \sqrt{a + x^2})^5} + \frac{2 a^3}{(-1 + n) (x - \sqrt{a + x^2})} + \frac{2 a^2 (x - \sqrt{a + x^2})}{1 + n} + \frac{(x - \sqrt{a + x^2})^5}{5 + n} + \frac{3 a^4}{(-3 + n) (-x + \sqrt{a + x^2})^3} + \frac{3 a (-x + \sqrt{a + x^2})^3}{3 + n} \right) + (4 a \sqrt{a + x^2} (2 a^3 n - 4 (3 - n - 3 n^2 + n^3) x^5 (-x + \sqrt{a + x^2}) + a^2 (-3 + n) n x ((-3 + n) x + 2 \sqrt{a + x^2}) - a (3 - 4 n + n^2) x^3 (- (3 + 5 n) x + (1 + 3 n) \sqrt{a + x^2}))) / ((-3 + n) (-1 + n) (1 + n) (3 + n) (x - \sqrt{a + x^2})^2 (-a + x (-x + \sqrt{a + x^2}))) \right)$$

Problem 493: Unable to integrate problem.

$$\int \frac{(x - \sqrt{a + x^2})^n}{a + x^2} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{2 (x - \sqrt{a + x^2})^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{(x - \sqrt{a + x^2})^2}{a}\right]}{a (1 + n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{a + x^2} dx$$

Problem 494: Unable to integrate problem.

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{8 (x - \sqrt{a + x^2})^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{(x - \sqrt{a + x^2})^2}{a}\right]}{a^3 (3 + n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Problem 495: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx$$

Optimal (type 3, 187 leaves, 3 steps):

$$\begin{aligned} & -\frac{a^6 (x + \sqrt{a + x^2})^{-6+n}}{64 (6 - n)} - \frac{3 a^5 (x + \sqrt{a + x^2})^{-4+n}}{32 (4 - n)} - \frac{15 a^4 (x + \sqrt{a + x^2})^{-2+n}}{64 (2 - n)} + \\ & \frac{5 a^3 (x + \sqrt{a + x^2})^n}{16 n} + \frac{15 a^2 (x + \sqrt{a + x^2})^{2+n}}{64 (2 + n)} + \frac{3 a (x + \sqrt{a + x^2})^{4+n}}{32 (4 + n)} + \frac{(x + \sqrt{a + x^2})^{6+n}}{64 (6 + n)} \end{aligned}$$

Result (type 3, 659 leaves):

$$\begin{aligned}
& \left((x + \sqrt{a+x^2})^{9+n} (a+x(x+\sqrt{a+x^2})) \left(\frac{4a^3}{n} + \frac{a^6}{(-6+n)(x+\sqrt{a+x^2})^6} - \right. \right. \\
& \quad \left. \left. \frac{2a^5}{(-4+n)(x+\sqrt{a+x^2})^4} - \frac{a^4}{(-2+n)(x+\sqrt{a+x^2})^2} - \frac{a^2(x+\sqrt{a+x^2})^2}{2+n} - \frac{2a(x+\sqrt{a+x^2})^4}{4+n} + \frac{(x+\sqrt{a+x^2})^6}{6+n} \right) \right) / \\
& \left(64 \left(512 x^{10} (x + \sqrt{a+x^2}) + a^5 (10 x + \sqrt{a+x^2}) + 256 a x^8 (6 x + 5 \sqrt{a+x^2}) + 10 a^4 x^2 (17 x + 5 \sqrt{a+x^2}) + \right. \right. \\
& \quad \left. \left. 16 a^3 x^4 (52 x + 25 \sqrt{a+x^2}) + 32 a^2 x^6 (53 x + 35 \sqrt{a+x^2}) \right) \right) + \\
& \left(2 a \sqrt{a+x^2} (x + \sqrt{a+x^2})^{4+n} \left(2 a^4 + a^3 (-4+n) x \left((-4+n) x - 2 \sqrt{a+x^2} \right) + 8 (-4+n) n x^7 (x + \sqrt{a+x^2}) + \right. \right. \\
& \quad \left. \left. 4 a (-4+n) n x^5 (4 x + 3 \sqrt{a+x^2}) + a^2 (-4+n) x^3 \left((-4+9n) x + 4 (-1+n) \sqrt{a+x^2} \right) \right) \right) / \left((-4+n) n (4+n) \right. \\
& \quad \left. \left(128 x^8 (x + \sqrt{a+x^2}) + a^4 (8 x + \sqrt{a+x^2}) + 64 a x^6 (5 x + 4 \sqrt{a+x^2}) + 8 a^3 x^2 (11 x + 4 \sqrt{a+x^2}) + 16 a^2 x^4 (17 x + 10 \sqrt{a+x^2}) \right) \right) + \\
& \left(a^2 (a+x^2) (x + \sqrt{a+x^2})^n \left(a^2 (-2+n^2) + 2 (-2+n) n x^3 (x + \sqrt{a+x^2}) + a (-2+n) x \left((2+3n) x + 2 (1+n) \sqrt{a+x^2} \right) \right) \right) / \\
& \left(n (-4+n^2) \left(a+x(x+\sqrt{a+x^2}) \right)^2 \right)
\end{aligned}$$

Problem 496: Result more than twice size of optimal antiderivative.

$$\int (a+x^2)^{3/2} (x+\sqrt{a+x^2})^n dx$$

Optimal (type 3, 131 leaves, 3 steps):

$$-\frac{a^4 (x + \sqrt{a+x^2})^{-4+n}}{16 (4-n)} - \frac{a^3 (x + \sqrt{a+x^2})^{-2+n}}{4 (2-n)} + \frac{3 a^2 (x + \sqrt{a+x^2})^n}{8 n} + \frac{a (x + \sqrt{a+x^2})^{2+n}}{4 (2+n)} + \frac{(x + \sqrt{a+x^2})^{4+n}}{16 (4+n)}$$

Result (type 3, 355 leaves):

$$\frac{1}{n} \sqrt{a+x^2} \left(x + \sqrt{a+x^2} \right)^n$$

$$\left(\left(\left(x + \sqrt{a+x^2} \right)^4 \left(2a^4 + a^3(-4+n)x \left((-4+n)x - 2\sqrt{a+x^2} \right) + 8(-4+n)nx^7 \left(x + \sqrt{a+x^2} \right) + 4a(-4+n)nx^5 \left(4x + 3\sqrt{a+x^2} \right) + a^2(-4+n)x^3 \left((-4+9n)x + 4(-1+n)\sqrt{a+x^2} \right) \right) \right) \right) / \left((-4+n)(4+n) \right)$$

$$\left(128x^8 \left(x + \sqrt{a+x^2} \right) + a^4 \left(8x + \sqrt{a+x^2} \right) + 64ax^6 \left(5x + 4\sqrt{a+x^2} \right) + 8a^3x^2 \left(11x + 4\sqrt{a+x^2} \right) + 16a^2x^4 \left(17x + 10\sqrt{a+x^2} \right) \right) +$$

$$\left(a\sqrt{a+x^2} \left(a^2(-2+n^2) + 2(-2+n)nx^3 \left(x + \sqrt{a+x^2} \right) + a(-2+n)x \left((2+3n)x + 2(1+n)\sqrt{a+x^2} \right) \right) \right) / \left((-4+n^2) \left(a+x \left(x + \sqrt{a+x^2} \right) \right)^2 \right)$$

Problem 499: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{\left(a+x^2 \right)^{3/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{4 \left(x + \sqrt{a+x^2} \right)^{2+n} \text{Hypergeometric2F1} \left[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a} \right]}{a^2 (2+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{\left(a+x^2 \right)^{3/2}} dx$$

Problem 500: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{\left(a+x^2 \right)^{5/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{16 \left(x + \sqrt{a+x^2} \right)^{4+n} \text{Hypergeometric2F1} \left[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a} \right]}{a^4 (4+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x + \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Problem 501: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} (x - \sqrt{a + x^2})^n dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\frac{a^6 (x - \sqrt{a + x^2})^{-6+n}}{64 (6 - n)} + \frac{3 a^5 (x - \sqrt{a + x^2})^{-4+n}}{32 (4 - n)} + \frac{15 a^4 (x - \sqrt{a + x^2})^{-2+n}}{64 (2 - n)} -$$

$$\frac{5 a^3 (x - \sqrt{a + x^2})^n}{16 n} - \frac{15 a^2 (x - \sqrt{a + x^2})^{2+n}}{64 (2 + n)} - \frac{3 a (x - \sqrt{a + x^2})^{4+n}}{32 (4 + n)} - \frac{(x - \sqrt{a + x^2})^{6+n}}{64 (6 + n)}$$

Result (type 3, 692 leaves):

$$\left((x - \sqrt{a + x^2})^{9+n} (a + x (x - \sqrt{a + x^2})) \left(\frac{4 a^3}{n} + \frac{a^6}{(-6 + n) (x - \sqrt{a + x^2})^6} - \right. \right.$$

$$\left. \left. \frac{2 a^5}{(-4 + n) (x - \sqrt{a + x^2})^4} - \frac{a^4}{(-2 + n) (x - \sqrt{a + x^2})^2} - \frac{a^2 (x - \sqrt{a + x^2})^2}{2 + n} - \frac{2 a (x - \sqrt{a + x^2})^4}{4 + n} + \frac{(x - \sqrt{a + x^2})^6}{6 + n} \right) \right) /$$

$$\left(64 (a^5 (-10 x + \sqrt{a + x^2}) + 512 x^{10} (-x + \sqrt{a + x^2}) + 10 a^4 x^2 (-17 x + 5 \sqrt{a + x^2}) + 256 a x^8 (-6 x + 5 \sqrt{a + x^2}) + \right.$$

$$\left. 16 a^3 x^4 (-52 x + 25 \sqrt{a + x^2}) + 32 a^2 x^6 (-53 x + 35 \sqrt{a + x^2}) \right) +$$

$$\left(2 a \sqrt{a + x^2} (x - \sqrt{a + x^2})^{4+n} (-2 a^4 + 8 (-4 + n) n x^7 (-x + \sqrt{a + x^2}) - a^3 (-4 + n) x ((-4 + n) x + 2 \sqrt{a + x^2}) + \right.$$

$$\left. 4 a (-4 + n) n x^5 (-4 x + 3 \sqrt{a + x^2}) + a^2 (-4 + n) x^3 ((4 - 9 n) x + 4 (-1 + n) \sqrt{a + x^2}) \right) / ((-4 + n) n (4 + n))$$

$$\left(a^4 (-8 x + \sqrt{a + x^2}) + 128 x^8 (-x + \sqrt{a + x^2}) + 8 a^3 x^2 (-11 x + 4 \sqrt{a + x^2}) + 64 a x^6 (-5 x + 4 \sqrt{a + x^2}) + 16 a^2 x^4 (-17 x + 10 \sqrt{a + x^2}) \right) +$$

$$\left(a^2 (a + x^2) (x - \sqrt{a + x^2})^n (-a^2 (-2 + n^2) + 2 (-2 + n) n x^3 (-x + \sqrt{a + x^2}) + a (-2 + n) x (-(2 + 3 n) x + 2 (1 + n) \sqrt{a + x^2})) \right) /$$

$$\left(n (-4 + n^2) (a + x (x - \sqrt{a + x^2}))^2 \right)$$

Problem 502: Result more than twice size of optimal antiderivative.

$$\int (a+x^2)^{3/2} (x-\sqrt{a+x^2})^n dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{a^4 (x-\sqrt{a+x^2})^{-4+n}}{16(4-n)} + \frac{a^3 (x-\sqrt{a+x^2})^{-2+n}}{4(2-n)} - \frac{3a^2 (x-\sqrt{a+x^2})^n}{8n} - \frac{a (x-\sqrt{a+x^2})^{2+n}}{4(2+n)} - \frac{(x-\sqrt{a+x^2})^{4+n}}{16(4+n)}$$

Result (type 3, 366 leaves):

$$\frac{1}{n} (x-\sqrt{a+x^2})^n \left(\left(\sqrt{a+x^2} (x-\sqrt{a+x^2})^4 \left(-2a^4 + 8(-4+n)nx^7(-x+\sqrt{a+x^2}) - a^3(-4+n)x \left((-4+n)x + 2\sqrt{a+x^2} \right) + 4a(-4+n)nx^5(-4x+3\sqrt{a+x^2}) + a^2(-4+n)x^3 \left((4-9n)x + 4(-1+n)\sqrt{a+x^2} \right) \right) \right) / \left((-4+n)(4+n) \right) \right. \\ \left. \left(a^4(-8x+\sqrt{a+x^2}) + 128x^8(-x+\sqrt{a+x^2}) + 8a^3x^2(-11x+4\sqrt{a+x^2}) + 64ax^6(-5x+4\sqrt{a+x^2}) + 16a^2x^4(-17x+10\sqrt{a+x^2}) \right) \right) + \\ \left(a(a+x^2) \left(-a^2(-2+n^2) + 2(-2+n)nx^3(-x+\sqrt{a+x^2}) + a(-2+n)x \left(-(2+3n)x + 2(1+n)\sqrt{a+x^2} \right) \right) \right) / \left((-4+n^2) \left(a+x(x-\sqrt{a+x^2}) \right)^2 \right) \right)$$

Problem 505: Unable to integrate problem.

$$\int \frac{(x-\sqrt{a+x^2})^n}{(a+x^2)^{3/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{4(x-\sqrt{a+x^2})^{2+n} \operatorname{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{(x-\sqrt{a+x^2})^2}{a}\right]}{a^2(2+n)}$$

Result (type 8, 27 leaves):

$$\int \frac{(x-\sqrt{a+x^2})^n}{(a+x^2)^{3/2}} dx$$

Problem 506: Unable to integrate problem.

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{16 (x - \sqrt{a + x^2})^{4+n} \operatorname{Hypergeometric2F1}\left[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{(x - \sqrt{a + x^2})^2}{a}\right]}{a^4 (4 + n)}$$

Result (type 8, 27 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Problem 507: Unable to integrate problem.

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 3, 365 leaves, 4 steps):

$$\frac{(d^2 - af^2)^5 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-5+n}}{32 e f^4 (5 - n)} - \frac{5 (d^2 - af^2)^4 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-3+n}}{32 e f^4 (3 - n)} + \frac{5 (d^2 - af^2)^3 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-1+n}}{16 e f^4 (1 - n)} +$$

$$\frac{5 (d^2 - af^2)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{16 e f^4 (1 + n)} - \frac{5 (d^2 - af^2) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{3+n}}{32 e f^4 (3 + n)} + \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{5+n}}{32 e f^4 (5 + n)}$$

Result (type 8, 58 leaves):

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 508: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 239 leaves, 4 steps):

$$\frac{(d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-3+n}}{8 e f^2 (3 - n)} - \frac{3 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{8 e f^2 (1 - n)} - \frac{3 (d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{8 e f^2 (1 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{3+n}}{8 e f^2 (3 + n)}$$

Result (type 8, 56 leaves):

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 509: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$\frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{2 e (1 - n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1 + n)}$$

Result (type 8, 35 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 510: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Optimal (type 5, 122 leaves, 2 steps):

$$\frac{2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right]}{e (d^2 - a f^2) (1+n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Problem 511: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^2} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{8 f^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{3+n} \operatorname{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right]}{e (d^2 - a f^2)^3 (3+n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^2} dx$$

Problem 512: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$\frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{2 e (1 - n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1 + n)}$$

Result (type 8, 35 leaves):

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Problem 513: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^2}{d^2 - a f^2}\right]}{e (d^2 - a f^2) (1 + n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Problem 514: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 297 leaves, 4 steps):

$$\begin{aligned} & - \frac{(d^2 - a f^2)^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-4+n}}{16 e f^3 (4 - n)} + \frac{(d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f^3 (2 - n)} + \\ & \frac{3 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{8 e f^3 n} - \frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f^3 (2 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{4+n}}{16 e f^3 (4 + n)} \end{aligned}$$

Result (type 8, 60 leaves):

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 515: Unable to integrate problem.

$$\int \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 171 leaves, 4 steps):

$$\begin{aligned} & - \frac{(d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f (2 - n)} - \frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{2 e f n} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f (2 + n)} \end{aligned}$$

Result (type 8, 60 leaves):

$$\int \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 516: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{e n}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Problem 517: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^{3/2}} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{4 f^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{2+n} \operatorname{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right]}{e (d^2 - a f^2)^2 (2+n)}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^{3/2}} dx$$

Problem 518: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Problem 519: Unable to integrate problem.

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 327 leaves, 5 steps):

$$\frac{(d^2 - a f^2)^2 \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f (2 - n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} + \frac{(d^2 - a f^2) \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{2 e f n \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} + \frac{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f (2 + n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}}$$

Result (type 8, 64 leaves):

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 520: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}} dx$$

Optimal (type 3, 93 leaves, 4 steps):

$$\frac{f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Result (type 8, 64 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}} dx$$

Problem 521: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2} \right)^{3/2}} dx$$

Optimal (type 5, 177 leaves, 4 steps):

$$\left(4 f^3 \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n} \operatorname{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right] \right) /$$

$$\left(e (d^2 - a f^2)^2 g (2+n) \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \right)$$

Result (type 8, 64 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\left(a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}\right)^{3/2}} dx$$

Problem 522: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}\right)^n}{\sqrt{\frac{a f^2 g + e g x (2 d + e x)}{f^2}}} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{e n \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Result (type 8, 62 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}\right)^n}{\sqrt{\frac{a f^2 g + e g x (2 d + e x)}{f^2}}} dx$$

Problem 523: Unable to integrate problem.

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Optimal (type 4, 191 leaves, 7 steps):

$$-\frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{b^2 e + a^2 f} \sqrt{c + d x^2}}{\sqrt{b^2 c + a^2 d} \sqrt{e + f x^2}}\right]}{\sqrt{b^2 c + a^2 d} \sqrt{b^2 e + a^2 f}} + \frac{\sqrt{-c} \sqrt{1 + \frac{d x^2}{c}} \sqrt{1 + \frac{f x^2}{e}} \operatorname{EllipticPi}\left[-\frac{b^2 c}{a^2 d}, \operatorname{ArcSin}\left[\frac{\sqrt{d} x}{\sqrt{-c}}\right], \frac{c f}{d e}\right]}{a \sqrt{d} \sqrt{c + d x^2} \sqrt{e + f x^2}}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Problem 524: Result more than twice size of optimal antiderivative.

$$\int \frac{e - 2 f x^2}{e^2 + 4 d f x^2 + 4 e f x^2 + 4 f^2 x^4} dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$-\frac{\operatorname{Log}\left[e - 2 \sqrt{-d} \sqrt{f} x + 2 f x^2\right]}{4 \sqrt{-d} \sqrt{f}} + \frac{\operatorname{Log}\left[e + 2 \sqrt{-d} \sqrt{f} x + 2 f x^2\right]}{4 \sqrt{-d} \sqrt{f}}$$

Result (type 3, 191 leaves):

$$-\frac{(-d - 2e + \sqrt{d} \sqrt{d + 2e}) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d + e - \sqrt{d} \sqrt{d + 2e}}}\right]}{\sqrt{d + e - \sqrt{d} \sqrt{d + 2e}}} - \frac{(d + 2e + \sqrt{d} \sqrt{d + 2e}) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d + e + \sqrt{d} \sqrt{d + 2e}}}\right]}{\sqrt{d + e + \sqrt{d} \sqrt{d + 2e}}}$$

$$\frac{2 \sqrt{2} \sqrt{d} \sqrt{d + 2e} \sqrt{f}}{2 \sqrt{2} \sqrt{d} \sqrt{d + 2e} \sqrt{f}}$$

Problem 525: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e - 2 f x^2}{e^2 - 4 d f x^2 + 4 e f x^2 + 4 f^2 x^4} dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$-\frac{\operatorname{Log}\left[e - 2 \sqrt{d} \sqrt{f} x + 2 f x^2\right]}{4 \sqrt{d} \sqrt{f}} + \frac{\operatorname{Log}\left[e + 2 \sqrt{d} \sqrt{f} x + 2 f x^2\right]}{4 \sqrt{d} \sqrt{f}}$$

Result (type 3, 233 leaves):

$$\frac{\frac{(-i d + 2 i e + \sqrt{d} \sqrt{-d+2e}) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d+e-i \sqrt{d} \sqrt{-d+2e}}}\right] - (i d - 2 i e + \sqrt{d} \sqrt{-d+2e}) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d+e+i \sqrt{d} \sqrt{-d+2e}}}\right]}{\sqrt{-d+e-i \sqrt{d} \sqrt{-d+2e}} - \sqrt{-d+e+i \sqrt{d} \sqrt{-d+2e}}}}{2 \sqrt{2} \sqrt{d} \sqrt{-d+2e} \sqrt{f}}$$

Problem 526: Result is not expressed in closed-form.

$$\int \frac{e - 4 f x^3}{e^2 + 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e + 2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves):

$$\frac{\operatorname{RootSum}\left[e^2 + 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \operatorname{Log}[x-\#1] + 4 f \operatorname{Log}[x-\#1] \#1^3}{2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \&\right]}{4 f}$$

Problem 527: Result is not expressed in closed-form.

$$\int \frac{e - 4 f x^3}{e^2 - 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e + 2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves):

$$\frac{\operatorname{RootSum}\left[e^2 - 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \operatorname{Log}[x-\#1] + 4 f \operatorname{Log}[x-\#1] \#1^3}{-2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \&\right]}{4 f}$$

Problem 528: Unable to integrate problem.

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 + 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e - 2f(-1+n)x^n}{e^2 + 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Problem 529: Unable to integrate problem.

$$\int \frac{e - 2f(-1+n)x^n}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e - 2f(-1+n)x^n}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Problem 532: Result is not expressed in closed-form.

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4ef\#1^2 + 4f^2\#1^4 + 4df\#1^6 \&, \frac{3e\text{Log}[x-\#1]\#1 + 2f\text{Log}[x-\#1]\#1^3}{e+2f\#1^2+3d\#1^4} \&\right]}{8f}$$

8 f

Problem 533: Result is not expressed in closed-form.

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4ef\sqrt{1^2} + 4f^2\sqrt{1^4} - 4df\sqrt{1^6} \&, \frac{3e\text{Log}[x-\sqrt{1}] + 2f\text{Log}[x-\sqrt{1}]\sqrt{1^3}}{e+2f\sqrt{1^2}-3d\sqrt{1^4}} \&\right]}{8f}$$

Problem 536: Result is not expressed in closed-form.

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 86 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4ef\sqrt{1^3} + 4df\sqrt{1^4} + 4f^2\sqrt{1^6} \&, \frac{-e\text{Log}[x-\sqrt{1}] + f\text{Log}[x-\sqrt{1}]\sqrt{1^3}}{3e\sqrt{1+4d\sqrt{1^2}}+6f\sqrt{1^4}} \&\right]}{2f}$$

Problem 537: Result is not expressed in closed-form.

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 - 4dfx^4 + 4f^2x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 86 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4 e f \sqrt{1^3} - 4 d f \sqrt{1^4} + 4 f^2 \sqrt{1^6} \&, \frac{-e \text{Log}[x-\sqrt{1}] + f \text{Log}[x-\sqrt{1}] \sqrt{1^3}}{3 e \sqrt{1-4 d \sqrt{1^2} + 6 f \sqrt{1^4}} \& \right]}{2 f}$$

Problem 542: Unable to integrate problem.

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 + 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 + 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 543: Unable to integrate problem.

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 547: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x (a c + b c x^2 + d \sqrt{a + b x^2})} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right]}{\sqrt{a}(ac^2-d^2)} + \frac{c \operatorname{Log}[x]}{ac^2-d^2} - \frac{c \operatorname{Log}[d+c\sqrt{a+bx^2}]}{ac^2-d^2}$$

Result (type 3, 282 leaves):

$$-\frac{1}{2ac^2-2d^2} \left(c \operatorname{Log}[4] + \left(-2c + \frac{2d}{\sqrt{a}} \right) \operatorname{Log}[x] + c \operatorname{Log}[ac^2-d^2+bc^2x^2] - \frac{2d \operatorname{Log}[a+\sqrt{a}\sqrt{a+bx^2}]}{\sqrt{a}} + \right. \\ \left. c \operatorname{Log}\left[\frac{(ac^2-d^2)(ac-i\sqrt{b}\sqrt{ac^2-d^2}x+d\sqrt{a+bx^2})}{\sqrt{b}cd^2(i\sqrt{ac^2-d^2}+\sqrt{b}cx)} \right] + c \operatorname{Log}\left[\frac{(ac^2-d^2)(ac+i\sqrt{b}\sqrt{ac^2-d^2}x+d\sqrt{a+bx^2})}{\sqrt{b}cd^2(-i\sqrt{ac^2-d^2}+\sqrt{b}cx)} \right] \right)$$

Problem 548: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3(ac+bcx^2+d\sqrt{a+bx^2})} dx$$

Optimal (type 3, 151 leaves, 8 steps):

$$-\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} - \frac{bd(3ac^2-d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right]}{2a^{3/2}(ac^2-d^2)^2} - \frac{bc^3 \operatorname{Log}[x]}{(ac^2-d^2)^2} + \frac{bc^3 \operatorname{Log}[d+c\sqrt{a+bx^2}]}{(ac^2-d^2)^2}$$

Result (type 3, 430 leaves):

$$\frac{1}{2a^{3/2}(-ac^2+d^2)^2x^2} \\ \left(-a^{5/2}c^3 + a^{3/2}cd^2 + a^{3/2}c^2d\sqrt{a+bx^2} - \sqrt{a}d^3\sqrt{a+bx^2} - b(2a^{3/2}c^3 - 3ac^2d + d^3)x^2 \operatorname{Log}[x] + a^{3/2}bc^3x^2 \operatorname{Log}[ac^2-d^2+bc^2x^2] - \right. \\ \left. 3abc^2dx^2 \operatorname{Log}[a+\sqrt{a}\sqrt{a+bx^2}] + bd^3x^2 \operatorname{Log}[a+\sqrt{a}\sqrt{a+bx^2}] + a^{3/2}bc^3x^2 \operatorname{Log}\left[-\frac{2(-ac^2+d^2)^2(ac-i\sqrt{b}\sqrt{ac^2-d^2}x+d\sqrt{a+bx^2})}{b^{3/2}c^3d^2(i\sqrt{ac^2-d^2}+\sqrt{b}cx)} \right] + \right. \\ \left. a^{3/2}bc^3x^2 \operatorname{Log}\left[-\frac{2(-ac^2+d^2)^2(ac+i\sqrt{b}\sqrt{ac^2-d^2}x+d\sqrt{a+bx^2})}{b^{3/2}c^3d^2(-i\sqrt{ac^2-d^2}+\sqrt{b}cx)} \right] \right)$$

Problem 555: Unable to integrate problem.

$$\int \frac{1}{x (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 3, 93 leaves, 7 steps):

$$\frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 \sqrt{a} (a c^2 - d^2)} + \frac{c \operatorname{Log}[x]}{a c^2 - d^2} - \frac{2 c \operatorname{Log}[d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$-\frac{a c - d \sqrt{a + b x^3}}{3 a (a c^2 - d^2) x^3} - \frac{b d (3 a c^2 - d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 a^{3/2} (a c^2 - d^2)^2} - \frac{b c^3 \operatorname{Log}[x]}{(a c^2 - d^2)^2} + \frac{2 b c^3 \operatorname{Log}[d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)^2}$$

Result (type 6, 596 leaves):

$$\frac{1}{9} \left(\left(6 b^2 c^2 d x^3 \operatorname{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(4 a (a c^2 - d^2) \operatorname{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + \right. \right. \right. \\ \left. \left. \left. b x^3 \left(-2 a c^2 \operatorname{AppellF1} \left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (-a c^2 + d^2) \operatorname{AppellF1} \left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) + \\ \frac{1}{a x^3} \left(- \left(\left(5 b^2 c^2 d (3 a c^2 - d^2) x^6 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3} \right] \right) / \right. \right. \\ \left. \left((a c^2 - d^2) \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(5 b c^2 x^3 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3} \right] + \right. \right. \right. \\ \left. \left. \left. (-2 a c^2 + 2 d^2) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3} \right] - a c^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3} \right] \right) \right) \right) + \\ \left. \left. \left. -3 (a c^2 - d^2) (a c - d \sqrt{a + b x^3}) - 9 a b c^3 x^3 \operatorname{Log}[x] + 3 a b c^3 x^3 \operatorname{Log}[a c^2 - d^2 + b c^2 x^3] \right) \right) \right) / \\ \left. \left. \left. (-a c^2 + d^2)^2 \right) \right) \right)$$

Problem 558: Unable to integrate problem.

$$\int \frac{x}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 304 leaves, 9 steps):

$$- \frac{d x^2 \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right]}{2 (a c^2 - d^2) \sqrt{a + b x^3}} - \frac{\operatorname{ArcTan} \left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3} b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} - \\ \frac{\operatorname{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} + \frac{\operatorname{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}}$$

Result (type 8, 29 leaves):

$$\int \frac{x}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Problem 559: Unable to integrate problem.

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 300 leaves, 9 steps):

$$\begin{aligned} & - \frac{d x \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) \sqrt{a + b x^3}} - \frac{c^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} (a c^2 - d^2)^{2/3}} + \\ & \frac{c^{1/3} \operatorname{Log}\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 b^{1/3} (a c^2 - d^2)^{2/3}} - \frac{c^{1/3} \operatorname{Log}\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 b^{1/3} (a c^2 - d^2)^{2/3}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Problem 560: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 319 leaves, 10 steps):

$$\begin{aligned} & - \frac{c}{(a c^2 - d^2) x} + \frac{d \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[-\frac{1}{3}, \frac{1}{2}, 1, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) x \sqrt{a + b x^3}} + \frac{b^{1/3} c^{5/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \\ & \frac{b^{1/3} c^{5/3} \operatorname{Log}\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{4/3}} - \frac{b^{1/3} c^{5/3} \operatorname{Log}\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{4/3}} \end{aligned}$$

Result (type 6, 1029 leaves):

$$\begin{aligned}
& -\frac{c}{a c^2 x - d^2 x} + \frac{d \sqrt{a + b x^3}}{a^2 c^2 x - a d^2 x} + \left(5 a b c^2 d x^2 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \\
& \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(10 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] - \right. \right. \\
& \quad \left. \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) \right) + \\
& \left(5 b d^3 x^2 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right) \\
& \left(10 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] - \right. \\
& \quad \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) + \\
& \left(8 b^2 c^2 d x^5 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \left(5 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right) \\
& \left(-16 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + \right. \\
& \quad \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) \right) - \\
& \frac{b^{1/3} c^{5/3} \operatorname{ArcTan}\left[\frac{-1 + \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \frac{b^{1/3} c^{5/3} \operatorname{Log}\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{4/3}} - \frac{b^{1/3} c^{5/3} \operatorname{Log}\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{4/3}}
\end{aligned}$$

Problem 561: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 324 leaves, 10 steps):

$$\begin{aligned}
& -\frac{c}{2 (a c^2 - d^2) x^2} + \frac{d \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{2 (a c^2 - d^2) x^2 \sqrt{a + b x^3}} + \frac{b^{2/3} c^{7/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a c^2 - d^2)^{5/3}} - \\
& \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{5/3}} + \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Result (type 6, 1044 leaves):

$$\begin{aligned}
& - \frac{c}{2(a^2 c^2 - d^2) x^2} + \frac{d \sqrt{a + b x^3}}{2 a (a c^2 - d^2) x^2} + \left(10 a b c^2 d x \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \\
& \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(8 a (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \quad \left. \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) - \\
& \left(2 b d^3 x \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(8 a (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \quad \left. \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) + \\
& \left(7 b^2 c^2 d x^4 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(8 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \quad \left. \left(14 a (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \quad \left. \left. 3 b x^3 \left(2 a c^2 \operatorname{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) - \\
& \frac{b^{2/3} c^{7/3} \operatorname{ArcTan} \left[\frac{-(a c^2 - d^2)^{1/3} + 2 b^{1/3} c^{2/3} x}{\sqrt{3} (a c^2 - d^2)^{1/3}} \right]}{\sqrt{3} (a c^2 - d^2)^{5/3}} - \frac{b^{2/3} c^{7/3} \operatorname{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 (a c^2 - d^2)^{5/3}} + \\
& \frac{b^{2/3} c^{7/3} \operatorname{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Problem 562: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$- \frac{d x \sqrt{1 + \frac{b x^n}{a}} \operatorname{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) \sqrt{a + b x^n}} + \frac{c x \operatorname{Hypergeometric2F1} \left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{a c^2 - d^2}$$

Result (type 6, 320 leaves):

$$\begin{aligned}
& - \left(\left(2 a d (a c^2 - d^2) (1+n) \operatorname{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) / \right. \\
& \quad \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \left(-2 a b c^2 n x^n \operatorname{AppellF1} \left[1 + \frac{1}{n}, \frac{1}{2}, 2, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + \right. \right. \\
& \quad \left. \left. (a c^2 - d^2) \left(-b n x^n \operatorname{AppellF1} \left[1 + \frac{1}{n}, \frac{3}{2}, 1, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + 2 a (1+n) \operatorname{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \right) \Bigg) + \\
& \quad \frac{c x \operatorname{Hypergeometric2F1} \left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{a c^2 - d^2}
\end{aligned}$$

Problem 563: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 167 leaves, 4 steps):

$$-\frac{d x^{1+m} \sqrt{1 + \frac{b x^n}{a}} \operatorname{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) (1+m) \sqrt{a + b x^n}} + \frac{c x^{1+m} \operatorname{Hypergeometric2F1} \left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) (1+m)}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& \frac{1}{(a c^2 - d^2) (1+m)} \\
& x^{1+m} \left(- \left(\left(2 a d (-a c^2 + d^2)^2 (1+m+n) \operatorname{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) / \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \left(2 a (a c^2 - d^2) \right. \right. \right. \\
& \quad \left. \left. (1+m+n) \operatorname{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] - b n x^n \left(2 a c^2 \operatorname{AppellF1} \left[\frac{1+m+n}{n}, \frac{1}{2}, 2, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a c^2 - d^2) \operatorname{AppellF1} \left[\frac{1+m+n}{n}, \frac{3}{2}, 1, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \right) \Bigg) + c \operatorname{Hypergeometric2F1} \left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]
\end{aligned}$$

Problem 566: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} - x^{5/2}} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\operatorname{ArcTan}[\sqrt{x}] + \operatorname{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 587: Unable to integrate problem.

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)^n dx$$

Optimal (type 6, 80 leaves, 4 steps):

$$\frac{\left(a + \frac{b}{x}\right)^m x \left(1 + \frac{ax}{b}\right)^{-m} (c + dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[1 - m, -m, -n, 2 - m, -\frac{ax}{b}, -\frac{dx}{c}\right]}{1 - m}$$

Result (type 8, 19 leaves):

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)^n dx$$

Problem 591: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{c + dx} dx$$

Optimal (type 5, 101 leaves, 5 steps):

$$-\frac{c \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{c \left(a + \frac{b}{x}\right)}{ac - bd}\right]}{d (ac - bd) (1 + m)} + \frac{\left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, 1 + \frac{b}{ax}\right]}{ad (1 + m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{c + dx} dx$$

Problem 592: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + dx)^2} dx$$

Optimal (type 5, 56 leaves, 3 steps):

$$\frac{b \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{(a c - b d)^2 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^2} dx$$

Problem 593: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Optimal (type 5, 112 leaves, 4 steps):

$$\frac{d \left(a + \frac{b}{x}\right)^{1+m}}{2 c (a c - b d) \left(d + \frac{c}{x}\right)^2} - \frac{b (2 a c - b d (1+m)) \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{2 c (a c - b d)^3 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Problem 594: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^4} dx$$

Optimal (type 5, 185 leaves, 5 steps):

$$\frac{d^2 \left(a + \frac{b}{x}\right)^{1+m}}{3 c^2 (a c - b d) \left(d + \frac{c}{x}\right)^3} - \frac{d (6 a c - b d (4+m)) \left(a + \frac{b}{x}\right)^{1+m}}{6 c^2 (a c - b d)^2 \left(d + \frac{c}{x}\right)^2} - \frac{b (6 a^2 c^2 - 6 a b c d (1+m) + b^2 d^2 (2+3 m+m^2)) \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{6 c^2 (a c - b d)^4 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + dx)^4} dx$$

Problem 598: Unable to integrate problem.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx$$

Optimal (type 3, 28 leaves, 3 steps):

$$\frac{\sqrt{b - \frac{a}{x^2}} x \operatorname{Log}[x]}{\sqrt{a - bx^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx$$

Problem 601: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c + dx)^{3/2}}{\sqrt{a + \frac{b}{x^2}}} dx$$

Optimal (type 4, 406 leaves, 8 steps):

$$\frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2\sqrt{b}(ac^2-3bd^2)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{-a}x}}{\sqrt{b}}}\right], -\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right]}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}}$$

$$\frac{2\sqrt{b}c(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\sqrt{1+\frac{ax^2}{b}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{-a}x}}{\sqrt{b}}}\right], -\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right]}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{c+dx}}$$

Result (type 4, 540 leaves):

$$\frac{1}{5\sqrt{a+\frac{b}{x^2}}x}\sqrt{c+dx}$$

$$\left(\frac{2(2c+dx)(b+ax^2)}{a} + 2\left(d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}\left(-3b^2d^2+a^2c^2x^2+ab(c^2-3d^2x^2)\right)+\sqrt{a}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/2}d^3\right)\right)\right)$$

$$\sqrt{\frac{d\left(\frac{i\sqrt{b}}{\sqrt{a}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{i\sqrt{b}d}{\sqrt{a}}-dx}{c+dx}}(c+dx)^{3/2}\text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}}{\sqrt{c+dx}}\right], \frac{\sqrt{a}c-i\sqrt{b}d}{\sqrt{a}c+i\sqrt{b}d}\right] -$$

$$\sqrt{a}\sqrt{b}d\left(ac^2+4i\sqrt{a}\sqrt{b}cd-3bd^2\right)\sqrt{\frac{d\left(\frac{i\sqrt{b}}{\sqrt{a}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{i\sqrt{b}d}{\sqrt{a}}-dx}{c+dx}}(c+dx)^{3/2}$$

$$\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}}{\sqrt{c+dx}}\right], \frac{\sqrt{a}c-i\sqrt{b}d}{\sqrt{a}c+i\sqrt{b}d}\right] \Bigg/ \left(a^2d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}(c+dx)\right)$$

Problem 681: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1 + \frac{1}{x^2}} x}{(1 + x^2)^2} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} x^2}{1 + x^2}$$

Problem 682: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + \frac{1}{x^2}} x (1 + x^2)} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} x^2}{1 + x^2}$$

Problem 737: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$2 \operatorname{ArcTan} \left[\sqrt{-\frac{x}{1+x}} \right]$$

Result (type 3, 32 leaves):

$$\frac{2 \sqrt{-\frac{x}{1+x}} \sqrt{1+x} \operatorname{ArcSinh}[\sqrt{x}]}{\sqrt{x}}$$

Problem 738: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$2 \operatorname{ArcTan} \left[\sqrt{\frac{1-x}{1+x}} \right]$$

Result (type 3, 47 leaves):

$$\frac{2 \sqrt{\frac{1-x}{1+x}} \sqrt{1-x^2} \operatorname{ArcSin} \left[\frac{\sqrt{1+x}}{\sqrt{2}} \right]}{-1+x}$$

Problem 739: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan} \left[\sqrt{\frac{a+bx}{c-bx}} \right]}{b}$$

Result (type 3, 80 leaves):

$$\frac{i \sqrt{c-bx} \sqrt{\frac{a+bx}{c-bx}} \operatorname{Log} \left[2 \sqrt{c-bx} \sqrt{a+bx} - i (a-c+2bx) \right]}{b \sqrt{a+bx}}$$

Problem 740: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 89 leaves):

$$\frac{\sqrt{\frac{a+bx}{c+dx}} \sqrt{c+dx} \operatorname{Log} \left[bc+ad+2bdx+2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx} \right]}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

Problem 759: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x + \sqrt{-3-4x-x^2}} dx$$

Optimal (type 3, 108 leaves, 10 steps):

$$-\operatorname{ArcTan} \left[\frac{\sqrt{-1-x}}{\sqrt{3+x}} \right] - \sqrt{2} \operatorname{ArcTan} \left[\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}} \right] + \frac{1}{2} \operatorname{Log} [3+x] + \frac{1}{2} \operatorname{Log} \left[\frac{3\sqrt{-1-x} + \sqrt{-1-x}x + x\sqrt{3+x}}{(3+x)^{3/2}} \right]$$

Result (type 3, 1012 leaves):

$$\begin{aligned}
& \frac{1}{8} \left(4 \operatorname{ArcSin}[2+x] - 4\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] + 2\sqrt{1-2i\sqrt{2}} \right. \\
& \operatorname{ArcTan} \left[\left(60 + 51i\sqrt{2} + (-16 + 6i\sqrt{2})x^4 + 54i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2} + x(68 + 176i\sqrt{2} + 99i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}) \right) \right. \\
& \left. \left. + 2ix^3 \left(34(i+\sqrt{2}) + 9\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2} \right) + ix^2 \left(44i + 185\sqrt{2} + 72\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2} \right) \right] \right) / \\
& \left(93i + 150\sqrt{2} + 20(17i + 22\sqrt{2})x + (493i + 466\sqrt{2})x^2 + 16(19i + 13\sqrt{2})x^3 + (66i + 32\sqrt{2})x^4 \right) - \frac{1}{\sqrt{1+2i\sqrt{2}}} 2i(-i+2\sqrt{2}) \\
& \operatorname{ArcTan} \left[\left(-60 + 51i\sqrt{2} + 2(8 + 3i\sqrt{2})x^4 + 54i\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} + 2x^3 \left(34 + 34i\sqrt{2} + 9i\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} \right) \right) \right. \\
& \left. \left. + x^2 \left(44 + 185i\sqrt{2} + 72i\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} \right) + ix \left(68i + 176\sqrt{2} + 99\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} \right) \right] \right) / \\
& \left(-93i + 150\sqrt{2} + 20(-17i + 22\sqrt{2})x + (-493i + 466\sqrt{2})x^2 + 16(-19i + 13\sqrt{2})x^3 + (-66i + 32\sqrt{2})x^4 \right) + \\
& 2 \operatorname{Log}[3+4x+2x^2] + \frac{(-i+2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2i\sqrt{2}}} + \frac{(i+2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2i\sqrt{2}}} - \frac{1}{\sqrt{1-2i\sqrt{2}}} \\
& \left. \left((i+2\sqrt{2}) \operatorname{Log}[(3+4x+2x^2) \left(3+6i\sqrt{2} + (2+2i\sqrt{2})x^2 - 2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2} + x(4+8i\sqrt{2} - 2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}) \right)] \right) \right) - \\
& \frac{1}{\sqrt{1+2i\sqrt{2}}} (-i+2\sqrt{2}) \\
& \left. \operatorname{Log}[(3+4x+2x^2) \left(3-6i\sqrt{2} + (2-2i\sqrt{2})x^2 - 2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2} - 2x(-2+4i\sqrt{2} + \sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}) \right)] \right) \Bigg]
\end{aligned}$$

Problem 760: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x + \sqrt{-3-4x-x^2})^2} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$\frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{3+x} + \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 881 leaves):

$$\begin{aligned}
& \frac{1}{16} \left(\frac{8(3+x)}{3+4x+2x^2} + \frac{8(3+2x)\sqrt{-3-4x-x^2}}{3+4x+2x^2} + 4\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] - \frac{1}{\sqrt{1+2i\sqrt{2}}} \right. \\
& 2i(-2i+\sqrt{2}) \operatorname{ArcTan} \left[\frac{(2+x)(3(5+4i\sqrt{2})+16(2+i\sqrt{2})x+2(9+2i\sqrt{2})x^2)}{(12i-6\sqrt{2}+(8i+6\sqrt{2})x^3 - \right. \\
& \left. \left. 9\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} + x(40i-5\sqrt{2}-12\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}) + x^2(36i+8\sqrt{2}-6\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}) \right)} \right] + \\
& \left. \frac{1}{\sqrt{1-2i\sqrt{2}}} 2(2i+\sqrt{2}) \operatorname{ArcTanh} \left[\frac{(2+x)(3(5i+4\sqrt{2})+16(2i+\sqrt{2})x+2(9i+2\sqrt{2})x^2)}{(-5(8i+\sqrt{2})x+(-8i+6\sqrt{2})x^3 - \right. \right. \\
& \left. \left. 12\sqrt{1-2i\sqrt{2}}x\sqrt{-3-4x-x^2} + x^2(-36i+8\sqrt{2}-6\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}) - 3(4i+2\sqrt{2}+3\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}) \right)} \right] - \right. \\
& \left. \frac{(-2i+\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2i\sqrt{2}}} - \frac{(2i+\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2i\sqrt{2}}} + \frac{1}{\sqrt{1-2i\sqrt{2}}} \right. \\
& \left. (2i+\sqrt{2}) \operatorname{Log} \left[(3+4x+2x^2) \left(3+6i\sqrt{2} + (2+2i\sqrt{2})x^2 - 2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2} + x(4+8i\sqrt{2} - 2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}) \right) \right] \right] + \\
& \left. \frac{1}{\sqrt{1+2i\sqrt{2}}} (-2i+\sqrt{2}) \operatorname{Log} \left[(3+4x+2x^2) \left(3-6i\sqrt{2} + (2-2i\sqrt{2})x^2 - 2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2} - 2x(-2+4i\sqrt{2} + \sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}) \right) \right] \right]
\end{aligned}$$

Problem 761: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x + \sqrt{-3-4x-x^2})^3} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$-\frac{13 - \frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18 \left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}} \right)} - \frac{2 \left(2 - \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right)}{9 \left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}} \right)^2} - \frac{3 \operatorname{ArcTan} \left[\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}} \right]}{2\sqrt{2}}$$

Result (type 3, 914 leaves):

$$\begin{aligned}
& \frac{1}{32} \left(\frac{8(-3+2x)}{(3+4x+2x^2)^2} - \frac{8(2+3x)}{3+4x+2x^2} - \frac{8\sqrt{-3-4x-x^2}(15+26x+22x^2+8x^3)}{(3+4x+2x^2)^2} - 12\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] + \frac{1}{\sqrt{1+2i\sqrt{2}}} \right. \\
& 6(2+i\sqrt{2}) \operatorname{ArcTan}\left[\left((2+x)\left(3(5+4i\sqrt{2})+16(2+i\sqrt{2})x+2(9+2i\sqrt{2})x^2\right)\right) / \left(12i-6\sqrt{2}+(8i+6\sqrt{2})x^3 - \right. \right. \\
& \left. \left. 9\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2} + x\left(40i-5\sqrt{2}-12\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right) + x^2\left(36i+8\sqrt{2}-6\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right] - \\
& \frac{1}{\sqrt{1-2i\sqrt{2}}} 6(2i+\sqrt{2}) \operatorname{ArcTanh}\left[\left((2+x)\left(3(5i+4\sqrt{2})+16(2i+\sqrt{2})x+2(9i+2\sqrt{2})x^2\right)\right) / \left(-5(8i+\sqrt{2})x + (-8i+6\sqrt{2})x^3 - \right. \right. \\
& \left. \left. 12\sqrt{1-2i\sqrt{2}}x\sqrt{-3-4x-x^2} + x^2\left(-36i+8\sqrt{2}-6\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right) - 3\left(4i+2\sqrt{2}+3\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right] + \\
& \frac{3(-2i+\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2i\sqrt{2}}} + \frac{3(2i+\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2i\sqrt{2}}} - \frac{1}{\sqrt{1-2i\sqrt{2}}} 3(2i+\sqrt{2}) \\
& \operatorname{Log}\left[\left(3+4x+2x^2\right)\left(3+6i\sqrt{2}+(2+2i\sqrt{2})x^2-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8i\sqrt{2}-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right] - \\
& \frac{1}{\sqrt{1+2i\sqrt{2}}} 3(-2i+\sqrt{2}) \\
& \left. \operatorname{Log}\left[\left(3+4x+2x^2\right)\left(3-6i\sqrt{2}+(2-2i\sqrt{2})x^2-2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4i\sqrt{2}+\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right]
\end{aligned}$$

Problem 764: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps):

$$\begin{aligned}
& \frac{2}{35} (13 - 3(-1+x)^2) \sqrt{3 - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{7} (3 - 2(-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\
& \frac{16}{5} \sqrt{3} \operatorname{EllipticE}[\operatorname{ArcSin}[1-x], -\frac{1}{3}] - \frac{176}{35} \sqrt{3} \operatorname{EllipticF}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]
\end{aligned}$$

Result (type 4, 278 leaves):

$$\left(\begin{array}{l}
 896 - 1056x + 352x^2 + 848x^3 - 1420x^4 + 1152x^5 - 602x^6 + \\
 206x^7 - 45x^8 + 5x^9 + \frac{112i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}} \\
 304i\sqrt{2}\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}x^2\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] \Big/ \left(35\sqrt{-x(-8+8x-4x^2+x^3)}\right)
 \end{array} \right)$$

Problem 765: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{2\operatorname{EllipticE}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}} - \frac{4\operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 256 leaves):

$$- \left(\frac{-16 + 24x - 24x^2 + 14x^3 - 5x^4 + x^5 - \frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}}}{\sqrt{2} \sqrt{i+\sqrt{3}-\frac{4i}{x}} \sqrt{-x(-8+8x-4x^2+x^3)}} + \frac{8i\sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\left(3\sqrt{-x(-8+8x-4x^2+x^3)}\right)} \right)$$

Problem 766: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 156 leaves):

$$\frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} x(-4+x-i\sqrt{3}x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\sqrt{2} \sqrt{i+\sqrt{3}-\frac{4i}{x}} \sqrt{-x(-8+8x-4x^2+x^3)}}$$

Problem 767: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$\frac{(5 + (-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{8\sqrt{3}} - \frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{4\sqrt{3}}$$

Result (type 4, 261 leaves):

$$\frac{1}{24(-2+x)x} \sqrt{-x(-8+8x-4x^2+x^3)} \left(\frac{\sqrt{2}(-i+\sqrt{3}) \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\sqrt{\frac{4-2x+x^2}{x^2}}}}{2+x^2-4i\sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{4-2x+x^2}} \right)$$

Problem 768: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{7 \operatorname{EllipticE}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{144\sqrt{3}} - \frac{11 \operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{144\sqrt{3}}$$

Result (type 4, 298 leaves):

$$\left(\frac{7 i \sqrt{2} (-2+x) x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{i+\sqrt{3}-\frac{4i}{x}}{\sqrt{2} 3^{1/4}}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right]}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}}} + \right. \\ \left. \frac{1}{-8+8x-4x^2+x^3} \left(36 - 232x + 274x^2 - 226x^3 + 115x^4 - 37x^5 + 7x^6 - 19i\sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} x^3 \sqrt{\frac{4-2x+x^2}{x^2}} \right. \right. \\ \left. \left. (-8+8x-4x^2+x^3) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{i+\sqrt{3}-\frac{4i}{x}}{\sqrt{2} 3^{1/4}}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] \right) \right) / \left(432x \sqrt{-x(-8+8x-4x^2+x^3)} \right)$$

Problem 769: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int ((2-x)x(4-2x+x^2))^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps):

$$\frac{2}{35} (13 - 3(-1+x)^2) \sqrt{3 - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{7} (3 - 2(-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\ \frac{16}{5} \sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right] - \frac{176}{35} \sqrt{3} \operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]$$

Result (type 4, 278 leaves):

$$\frac{1}{35(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}}$$

$$\sqrt{-x(-8+8x-4x^2+x^3)} \left(\sqrt{\frac{4-2x+x^2}{x^2}} (-224+152x+44x^2-228x^3+230x^4-116x^5+35x^6-5x^7) + 112\sqrt{2}(-i+\sqrt{3}) \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} \right.$$

$$\left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] + 304i\sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] \right)$$

Problem 770: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{(2-x)x(4-2x+x^2)} dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\frac{1}{3} \sqrt{3-2(-1+x)^2-(-1+x)^4} (-1+x) + \frac{2 \text{EllipticE}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}} - \frac{4 \text{EllipticF}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 256 leaves):

$$\frac{1}{3(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}} \sqrt{-x(-8+8x-4x^2+x^3)}$$

$$\left(\sqrt{\frac{4-2x+x^2}{x^2}} (-4+4x-3x^2+x^3) + 2\sqrt{2}(-i+\sqrt{3}) \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] + \right.$$

$$\left. 8i\sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}\right] \right)$$

Problem 771: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 100 leaves):

$$\frac{(-1)^{1/3} (-2+x)^2 \sqrt{\frac{x(-1+i\sqrt{3}+x)}{(-2+x)^2}} \sqrt{\frac{-2+x-(-1)^{1/3}x}{-2+x}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right], (-1)^{2/3}\right]}{\sqrt{-x(-8+8x-4x^2+x^3)}}$$

Problem 772: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$\frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{\text{EllipticE}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{8\sqrt{3}} - \frac{\text{EllipticF}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{4\sqrt{3}}$$

Result (type 4, 298 leaves):

$$\left((-2+x)^2 x (4-2x+x^2) \left(2(-1+x)x - 3(4-2x+x^2) - \frac{3x(4-2x+x^2)}{-2+x} - \right. \right.$$

$$4(2-x) \sqrt{\frac{4-2x+x^2}{(-2+x)^2}} \left(x \sqrt{\frac{4-2x+x^2}{(-2+x)^2}} - \sqrt{2} (i+\sqrt{3}) \sqrt{\frac{ix}{(i+\sqrt{3})(-2+x)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{i+\sqrt{3}}\right] + \right.$$

$$\left. \left. \left. 4i\sqrt{2} \sqrt{\frac{ix}{(i+\sqrt{3})(-2+x)}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{i+\sqrt{3}}\right] \right) \right) \right) / (96(-x(-8+8x-4x^2+x^3))^{3/2})$$

Problem 773: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{7\operatorname{EllipticE}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{144\sqrt{3}} - \frac{11\operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{144\sqrt{3}}$$

Result (type 4, 327 leaves):

$$\frac{1}{7} \left(\frac{c}{d} + x \right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4 \right)^{3/2} +$$

$$\frac{2c \left(\frac{c}{d} + x \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \left(7c^3 + 20ad^2 - 3cd^2 \left(\frac{c}{d} + x \right)^2 \right)}{35d^2} - \frac{16c^3 (c^3 + 8ad^2) \left(\frac{c}{d} + x \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}{35d^2 \sqrt{c^3 + 4ad^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)} +$$

$$\left(16c^{13/4} (c^3 + 4ad^2)^{3/4} (c^3 + 8ad^2) \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right) \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}} \right) \right] \right) / \left(35d^5 \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) +$$

$$\left(8c^{7/4} (c^3 + 4ad^2)^{3/4} \left(\sqrt{c^3 + 4ad^2} (c^3 + 5ad^2) - c^{3/2} (c^3 + 8ad^2) \right) \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right) \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}} \right) \right] \right) / \left(35d^5 \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) +$$

Result (type 4, 10468 leaves):

$$\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \left(\frac{4c^2 (2c^3 + 15ad^2)}{35d^3} - \frac{4c (c^3 - 15ad^2)x}{35d^2} + \frac{2c^3x^2}{35d} + \frac{34c^2x^3}{35} + \frac{5}{7}cdx^4 + \frac{d^2x^5}{7} \right) +$$

$$\frac{1}{35d^3} 16c^2 \left(\left(2ac^3d \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{\frac{\left(\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \right) \right)$$

$$\begin{aligned}
& \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}} \\
& \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}}\right], \right. \\
& \left. \frac{\left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)} \right] / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right. \\
& \left. \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) + \\
& \left(40a^2d^3 \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}} \right)
\end{aligned}$$

$$\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right], \right.$$

$$\left. \frac{\left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)} \right] /$$

$$\left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) - \left(8c^5 \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right)$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right] \right), \\
& \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} + \frac{1}{d} \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}} \right], \\
& \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] \left/ \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(64 a c^2 d^2 \left(\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right)^2 \\
& \sqrt{\frac{\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + dx \right)}{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - dx \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + dx \right)}{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - dx \right)}} \right] \right), \\
& \frac{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right)^2} + \frac{1}{d} \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d}} \right], \\
& \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + dx \right)}{\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - dx \right)}} \right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right) \right) / \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right. \\
& \left. \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) - \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} \\
& c^4 d \left(\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) + \right. \\
& \left. 2 \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \right) \left(\frac{1}{2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}} d \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx\right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx\right)}} \right] \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right] + \left(d \frac{\left(\left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}\right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}\right)}{d} \right)}{d} \right) - \\
& \left. \frac{\left(-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}\right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}\right)}{d} \right) \text{EllipticF} \left[\right. \\
& \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx\right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx\right)}} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right] \left/ \left(2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) - \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) - \\
& \left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}} \right],
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right) / \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \Bigg) - \\
& \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} 8acd^3 \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) + 2 \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \right)
\end{aligned}$$

$$\left(\frac{1}{2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}} d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}} \left(\frac{c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx}{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx} \right)}\right], \right.$$

$$\left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right] + \left(d \frac{\left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}\right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}\right)}{d} \right)$$

$$\left. \frac{\left(-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}\right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}\right)}{d} \right) \text{EllipticF}\left[\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}} \left(\frac{c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx}{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx} \right)}\right],$$

$$\left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right] / \left(2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right)$$

$$\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) - \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right)$$

$$\left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}, \right.$$

$$\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right],$$

$$\left. \left. \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] / \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) \right)$$

Problem 775: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} dx$$

Optimal (type 4, 622 leaves, 5 steps):

$$\frac{1}{3} \left(\frac{c}{d} + x \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} - \frac{2c^2 \left(\frac{c}{d} + x \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}{3\sqrt{c^3 + 4ad^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)} +$$

$$\left(2c^{9/4} (c^3 + 4ad^2)^{3/4} \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)} \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}} \right) \right] \right) / \left(3d^3 \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) +$$

$$\left(c^{3/4} (c^3 + 4ad^2)^{1/4} \left(c^3 + 4ad^2 - c^{3/2} \sqrt{c^3 + 4ad^2} \right) \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)} \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}} \right) \right] \right) / \left(3d^3 \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right)$$

Result (type 4, 5218 leaves):

$$\left(\frac{c}{3d} + \frac{x}{3} \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} +$$

$$\frac{1}{3d} 2c \left(\left(8ad \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{\frac{\left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)} \right) \right)$$

$$\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right], \right.$$

$$\left. \frac{\left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)} \right] /$$

$$\left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) - \left(8c^2 \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right)$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right] \right), \\
& \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} + \frac{1}{d} 2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}} \right], \\
& \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] \left/ \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} cd \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) + \right. \\
& 2 \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \\
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \left(\frac{1}{2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}} d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \right. \right. \\
& \left. \left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right] \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] + d \left(\frac{\left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{d} - \right.
\end{aligned}$$

$$\left. \frac{\left(-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}\right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}\right)}{d} \right) \text{EllipticF}\left[
\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx\right)}}{\sqrt{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx\right)}} \right],$$

$$\left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right] \left/ \left(2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right. \right.$$

$$\left. \left. \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) - \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right. \right. \right.$$

$$\left. \left. \left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}, \right. \right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx\right)}}{\sqrt{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx\right)}} \right],$$

$$\left. \left. \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}\right)^2} \right) \right/ \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) \right) \right)$$

Problem 776: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} dx$$

Optimal (type 4, 227 leaves, 2 steps):

$$\left((c^3 + 4ad^2)^{1/4} \sqrt{\frac{d^2(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4ad^2}} \right) \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}} \right) \right] \right) / \left(2c^{1/4} d \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right)$$

Result (type 4, 822 leaves):

$$\left(2 \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right) \sqrt{-\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \right. \\ \left. \sqrt{-\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \text{EllipticF} \left[\right. \right. \\ \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right], \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] \right) / \left(d \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \sqrt{4ac + x^2 (2c + dx)^2} \right)$$

Problem 777: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{3/2}} dx$$

Optimal (type 4, 674 leaves, 5 steps):

$$\begin{aligned} & -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{8ac \left(c^3 + 4ad^2\right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} - \frac{d^2 \left(\frac{c}{d} + x\right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}{8a \left(c^3 + 4ad^2\right)^{3/2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4ad^2}}\right)} + \\ & \left(c^{1/4} \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4ad^2}}\right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4ad^2}}\right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}}\right)\right] \right) / \\ & \left(8ad (c^3 + 4ad^2)^{1/4} \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) + \left((c^3 + 4ad^2 - c^{3/2} \sqrt{c^3 + 4ad^2}) \sqrt{\frac{d^2 (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)}{(c^3 + 4ad^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4ad^2}}\right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4ad^2}}\right) \right) \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c + dx}{c^{1/4} (c^3 + 4ad^2)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4ad^2}}\right)\right] \right) / \left(16ac^{5/4}d (c^3 + 4ad^2)^{3/4} \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) \end{aligned}$$

Result (type 4, 5276 leaves):

$$\begin{aligned} & \frac{4acd + 2c^3x + 4ad^2x + 3c^2dx^2 + cd^2x^3}{8ac \left(c^3 + 4ad^2\right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} + \frac{1}{8ac \left(c^3 + 4ad^2\right)} \\ & d \left(\left(8ad \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{\frac{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)} \right) \right) \end{aligned}$$

$$\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right], \right.$$

$$\left. \frac{\left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)} \right] /$$

$$\left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) - \left(8c^2 \left(\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right.$$

$$\left. \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}}\right)$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right] \right), \\
& \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} + \frac{1}{d} 2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}}{-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d}} \right], \\
& \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] \left/ \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} \right) \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}} cd \left(\left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right) + \right. \\
& 2 \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)^2 \\
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \left(-\frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)}} \left(\frac{1}{2\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}} d \left(-\frac{-c - \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} + \right. \right. \\
& \left. \left. \frac{-c - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + dx \right)}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right) \left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - dx \right)} \right] \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2}{\left(\sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} - \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d} \right)^2} \right] + d \left(\frac{\left(-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d} \right) \left(-\frac{-c + \sqrt{c^2 - 2i\sqrt{a}\sqrt{c}d}}{d} - \frac{-c + \sqrt{c^2 + 2i\sqrt{a}\sqrt{c}d}}{d} \right)}{d} - \right.
\end{aligned}$$

$$\int \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} dx$$

Optimal (type 4, 663 leaves, 5 steps):

$$\frac{1}{3} \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} - \frac{2 d^2 \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{\sqrt{5 d^4 + 256 a e^3} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} +$$

$$\left(d^2 (5 d^4 + 256 a e^3)^{3/4} \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) / \left(8 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) +$$

$$\left((5 d^4 + 256 a e^3)^{1/4} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3} \right) \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) / \left(48 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 7543 leaves):

$$\left(\frac{d}{12e} + \frac{x}{3} \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} + \frac{1}{24e}$$

$$\left(\left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \sqrt{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \right. \\ \left. \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \right)$$

$$\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)^2 \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \\ \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)}$$

$$\sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \\ \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)}{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \right], \right]$$

$$\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \Bigg] / \\
\left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
\left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) + \left(256ae^3 \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
\sqrt{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right) \\
\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right) \\
\left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)^2 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right) \\
\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}$$

$$\sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\ \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}\right], \right]$$

$$\left(\frac{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)} \right) /$$

$$\left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\ \left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) -$$

$$\left(12d^3e \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)^2 \right)$$

$$\sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}}$$

$$\sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}}$$

$$\sqrt{\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}}$$

$$\left(-\frac{1}{4e} \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right)} \right] \right) \right)$$

$$\left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \Big/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right)$$

$$\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \Big] \Bigg], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \Big] + \frac{1}{2e}$$

$$\begin{aligned}
& \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \operatorname{EllipticPi} \left[\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}, \operatorname{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \right. \right. \right.} \right. \\
& \left. \left. \left. \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right] / \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \\
& \left. \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right] / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) - \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} \\
& 24d^2e^2 \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \left(\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) + \right. \\
& \left. \frac{1}{2} \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \right)
\end{aligned}$$

$$\sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}}$$

$$\sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}}$$

$$\sqrt{\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}}$$

$$\left(2e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right)} \right] \right) \right] \left. \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) \right]$$

$$\begin{aligned}
& \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) + \left(2e \left(\frac{\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} \right) \right. \\
& \left. \frac{\left(-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \right. \\
& \left. \left. \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right] \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right] \left. \right) \\
& \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) - \\
& \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& - \left(\left(\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \left(d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \right. \\
& \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}} \\
& \sqrt{\left(\left(\left(3d^2 - 2\sqrt{d^4 - 64ae^3} - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + \right. \right. \right. \\
& \left. \left. \left. d \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) + 4e \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right] / \left(2e \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right) \\
& \left. \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)} \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} \right)
\end{aligned}$$

Problem 780: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)^{3/2}} dx$$

Optimal (type 4, 748 leaves, 5 steps):

$$\frac{4 e \left(\frac{d}{4 e} + x\right) \left(13 d^4 - 256 a e^3 - 48 d^2 e^2 \left(\frac{d}{4 e} + x\right)^2\right)}{(5 d^8 - 64 a d^4 e^3 - 16 384 a^2 e^6) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} + \frac{384 d^2 e^2 \left(\frac{d}{4 e} + x\right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{(d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{3/2} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)} -$$

$$\left(12 \sqrt{2} d^2 \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}}\right)\right] \right) /$$

$$\left((d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{1/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) -$$

$$\left(2 \sqrt{2} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3}\right) \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right) \right) /$$

$$\left(\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}}\right)\right] \right) / \left((d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{3/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 7629 leaves):

$$\frac{2 (-5 d^5 + 128 a d e^3 - 8 d^4 e x + 512 a e^4 x + 72 d^3 e^2 x^2 + 96 d^2 e^3 x^3)}{(-d^4 + 64 a e^3) (5 d^4 + 256 a e^3) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} - \frac{1}{(d^4 - 64 a e^3) (5 d^4 + 256 a e^3)} 8 e$$

$$\left(\left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \sqrt{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \right)$$

$$\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)^2 \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)$$

$$\sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)}{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + X \right)} \right], \right]$$

$$\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
 \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \Bigg] / \\
 \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
 \left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) + \left(256ae^3 \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
 \sqrt{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\
 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\
 \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)^2 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\
 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}$$

$$\begin{aligned}
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)} \right], \right. \\
& \left. \left(\frac{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)} \right) \right] / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
& \left(\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) - \left(12d^3e \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}}$$

$$\sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}}$$

$$\sqrt{\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}}$$

$$\left(-\frac{1}{4e} \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right.} \right. \right. \right. \right. \\ \left. \left. \left. \left. \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right] \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right] + \frac{1}{2e}$$

$$\begin{aligned}
& \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \operatorname{EllipticPi} \left[\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}, \operatorname{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \right. \right. \right.} \right. \\
& \left. \left. \left. - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right] \right] / \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \\
& \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \left. \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right] / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) - \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} \\
& 24d^2e^2 \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \left(\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) + \right. \\
& \left. \frac{1}{2} \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}} \\
& \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}{\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + X \right)}} \\
& \sqrt{\frac{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right)}{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right)}} \\
& \left(2e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \text{EllipticE} \left[\right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{\frac{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right)}{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right)}} \right] \right],
\end{aligned}$$

$$\left(\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) +$$

$$\left(2e \frac{\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} \right) -$$

$$\left(\frac{\left(-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} \right) \Bigg) \text{EllipticF}[$$

$$\text{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \right) / \right.$$

$$\left. \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right) \Bigg],$$

$$\left(\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right.$$

$$\left. \left. \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) - \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)$$

$$\begin{aligned}
& \left. \left. \left. \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \text{EllipticPi} \left[\frac{-\frac{d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{-\frac{d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}, \right. \right. \\
& \text{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right) \right], \\
& \left. \left. \left. \left. \left. \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) / \left(-\frac{d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 781: Result more than twice size of optimal antiderivative.

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 452 leaves, 8 steps):

$$\begin{aligned}
& - \frac{16(7+2a)(1-\sqrt{4+a})\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \\
& \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{16(7+2a)(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \\
& \frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

Result (type 4, 6287 leaves):

$$\begin{aligned}
& \sqrt{a+8x-8x^2+4x^3-x^4}\left(\frac{1}{7}(-4-3a) + \frac{1}{35}(-32+15a)x + \frac{14x^2}{5} - \frac{66x^3}{35} + \frac{5x^4}{7} - \frac{x^5}{7}\right) + \\
& \frac{4}{35}\left(\left(40\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right.\right. \\
& \left.\left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right.\right. \\
& \left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(46a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(10a^2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(112\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(-1-\sqrt{-1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right],\right. \\
& \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + 2\sqrt{-1-\sqrt{4+a}} \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}\right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(32 a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + 2 \sqrt{-1-\sqrt{4+a}} \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
& 28 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}}} \right. \\
& \left. \left(\frac{1}{2 \sqrt{-1 - \sqrt{4+a}}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right) + \left(\left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \right. \\
& \left. \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right), \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 8a \left(\left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \\
& \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right], \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) + \\
& \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 782: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 397 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{1}{3} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{2 \left(1 - \sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{2(3+a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}
\end{aligned}$$

Result (type 4, 3470 leaves):

$$\begin{aligned}
& \left(-\frac{1}{3} + \frac{x}{3}\right) \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \frac{2}{3} \left(\left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}\right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(2a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right] \right], \right. \\
& \left. \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \Big/ \right. \\
& \left. \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \right. \\
& \left. \left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}}} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(\left(-1-\sqrt{-1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right]} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right]}, \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right) \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right) \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right) + \right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right)
\end{aligned}$$

$$\left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right.$$

$$\left. \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(2\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) +$$

$$\left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right) \right)$$

Problem 783: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} dx$$

Optimal (type 4, 144 leaves, 3 steps):

$$\frac{\sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

Result (type 4, 540 leaves):

$$\left(2 \left(1 + \sqrt{-1-\sqrt{4+a}} - x\right) \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(1 + \sqrt{-1+\sqrt{4+a}} - x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x\right)}} \right. \\ \left. \left(-1 + \sqrt{-1-\sqrt{4+a}} + x\right) \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x\right)}{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x\right)}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \\ \left(\sqrt{-1-\sqrt{4+a}} \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x\right)}} \sqrt{a-x(-8+8x-4x^2+x^3)} \right)$$

Problem 784: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$$

Optimal (type 4, 437 leaves, 7 steps):

$$\frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{(1-\sqrt{4+a})\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} +$$

$$\frac{(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}}} +$$

$$\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}}}$$

Result (type 4, 3526 leaves):

$$\frac{(6+a-8x-ax+3x^2-x^3)\sqrt{a+8x-8x^2+4x^3-x^4}}{2(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)} +$$

$$\frac{1}{2(3+a)(4+a)} \left(\left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}} \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}} \right], \right.$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(2a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(\left(-1-\sqrt{-1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right]} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2} \right] + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right]}, \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2} \right] \Bigg) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right) \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right) \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right) + \right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right)
\end{aligned}$$

$$\left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right.$$

$$\left. \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \left/ \left(2\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right) +$$

$$\left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \left/ \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right) \right) \right)$$

Problem 785: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 517 leaves, 8 steps):

$$\frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} -$$

$$\frac{(7+2a)(1-\sqrt{4+a})\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(7+2a)(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3(3+a)^2(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}}$$

$$\frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}}$$

Result (type 4, 6386 leaves):

$$\frac{\sqrt{a+8x-8x^2+4x^3-x^4}}{12(3+a)^2(4+a)^2} \left(\frac{-6-a+8x+ax-3x^2+x^3}{6(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)^2} + \frac{132+55a+5a^2-188x-71ax-5a^2x+84x^2+24ax^2-28x^3-8ax^3}{12(3+a)^2(4+a)^2(-a-8x+8x^2-4x^3+x^4)} \right) +$$

$$\left(\left(40 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)} \right], \right.$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(46a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \sqrt{a+8x-8x^2+4x^3-x^4} + \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(10a^2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(112\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right) \\
& \left(-1-\sqrt{-1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right],\right. \\
& \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + 2\sqrt{-1-\sqrt{4+a}} \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}\right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(32 a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + 2 \sqrt{-1-\sqrt{4+a}} \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
& 28 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \\
& \left. \frac{1}{2 \sqrt{-1 - \sqrt{4+a}}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right] \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \right. \\
& \left. \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right] \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 8a \left(\left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \\
& \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right], \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) + \\
& \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Big/ \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 786: Result more than twice size of optimal antiderivative.

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 558 leaves, 14 steps):

$$\begin{aligned} & \frac{3}{16} (4+a) (1+(-1+x)^2) \sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{8} (1+(-1+x)^2) (3+a-2(-1+x)^2-(-1+x)^4)^{3/2} - \\ & \frac{16(7+2a)(1-\sqrt{4+a})(1+\frac{(-1+x)^2}{1-\sqrt{4+a}})(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35} (13+5a-3(-1+x)^2) \sqrt{3+a-2(-1+x)^2-(-1+x)^4} (-1+x) + \\ & \frac{1}{7} (3+a-2(-1+x)^2-(-1+x)^4)^{3/2} (-1+x) + \frac{3}{16} (4+a)^2 \operatorname{ArcTan}\left[\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right] + \\ & \frac{16(7+2a)(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}(1+\frac{(-1+x)^2}{1-\sqrt{4+a}})\operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \\ & \frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}(1+\frac{(-1+x)^2}{1-\sqrt{4+a}})\operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \end{aligned}$$

Result (type 4, 7235 leaves):

$$\begin{aligned} & \frac{1}{a+8x-8x^2+4x^3-x^4} \left(\frac{1}{56} (52+11a) - \frac{1}{280} (116+55a)x + \frac{1}{80} (-36+25a)x^2 + \frac{74x^3}{35} - \frac{43x^4}{28} + \frac{17x^5}{28} - \frac{x^6}{8} \right) (a-x(-8+8x-4x^2+x^3))^{3/2} + \\ & \frac{1}{280(a+8x-8x^2+4x^3-x^4)^{3/2}} (a-x(-8+8x-4x^2+x^3))^{3/2} \end{aligned}$$

$$\begin{aligned} & \left(- \left(\left(2080 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right. \right. \\ & \left. \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left.\left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)-\right. \\
& \left.\left(208a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}\right)}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(110 a^2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}\right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(6944 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Big] + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2704 a \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}}} \right) \\
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Big] + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(210 a^2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} + 2 \sqrt{-1-\sqrt{4+a}} \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
& 896 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4+a}}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] + \right. \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2}{\left(\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2} \right] / \left(2\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) + \\
& \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right] \right), \right. \\
& \left. \frac{\left(\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2}{\left(\frac{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \Bigg) - \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 256a \left(\left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right) + \right. \\
& 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \\
& \left(\left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)+\left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \Big/ \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) + \\
& \left(4\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \Big/ \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \Big) \Big) \Big)
\end{aligned}$$

Problem 787: Result more than twice size of optimal antiderivative.

$$\int x \sqrt{a+8x-8x^2+4x^3-x^4} dx$$

Optimal (type 4, 466 leaves, 12 steps):

$$\frac{1}{4} \left(1 + (-1+x)^2\right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} - \frac{2(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{1}{3} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) +$$

$$\frac{1}{4} (4+a) \operatorname{ArcTan}\left[\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}\right] + \frac{2(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} +$$

$$\frac{2(3+a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}$$

Result (type 4, 4389 leaves):

$$\left(\frac{1}{6} - \frac{x}{6} + \frac{x^2}{4}\right) \sqrt{a-x(-8+8x-4x^2+x^3)} + \frac{1}{6\sqrt{a+8x-8x^2+4x^3-x^4}} \sqrt{a-x(-8+8x-4x^2+x^3)}$$

$$\left(- \left(\left(8 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)} \right], \right.$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left. \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \right. \\
& \left(2a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} + 2 \sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(6a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left((-1 - \sqrt{-1 - \sqrt{4+a}}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 4 \left((-1 + \sqrt{-1 - \sqrt{4+a}} + x) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \right. \\
& \left. \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 788: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 179 leaves, 7 steps):

$$\frac{1}{2} \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \right] + \frac{\sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}$$

Result (type 4, 865 leaves):

$$\begin{aligned}
& \left(2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + 2 \sqrt{-1-\sqrt{4+a}} \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Bigg) \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a-x} \left(-8+8x-4x^2+x^3 \right) \right)
\end{aligned}$$

Problem 789: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$$

Optimal (type 4, 474 leaves, 10 steps):

$$\frac{1 + (-1+x)^2}{2(4+a)\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} -$$

$$\frac{(1-\sqrt{4+a})\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

$$\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

Result (type 4, 3593 leaves):

$$\frac{(-a-2x+ax-ax^2-x^3)(a+8x-8x^2+4x^3-x^4)^2}{2(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)(a-x(-8+8x-4x^2+x^3))^{3/2}} + \frac{1}{2(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))^{3/2}}(a+8x-8x^2+4x^3-x^4)^{3/2}$$

$$\left(\left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)} \right], \right.$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(2a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right] \right], \right. \\
& \left. \left. \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \right. \right. \\
& \left. \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \right. \\
& \left. \left(4 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(\left(-1-\sqrt{-1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right]\right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}\right], \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}, \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\right) / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)+\right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}
\end{aligned}$$

$$\left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right.$$

$$\left. \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(2\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) +$$

$$\left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right.$$

$$\left. \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right) \right)$$

Problem 790: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 591 leaves, 12 steps):

$$\begin{aligned}
 & \frac{1 + (-1+x)^2}{6(4+a) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2}} + \frac{1 + (-1+x)^2}{3(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
 & \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2}} + \frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} - \\
 & \frac{(7+2a)(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(7+2a)(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3(3+a)^2(4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
 & \frac{(16+5a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12(3+a)(4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}
 \end{aligned}$$

Result (type 4, 6452 leaves):

$$\begin{aligned}
 & \frac{(a+8x-8x^2+4x^3-x^4)^3 \left(\frac{a+2x-ax+ax^2+x^3}{6(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)^2} + \frac{60+7a-3a^2-116x-23ax+3a^2x+48x^2-4a^2x^2-28x^3-8ax^3}{12(3+a)^2(4+a)^2(-a-8x+8x^2-4x^3+x^4)} \right)}{(a-x(-8+8x-4x^2+x^3))^{5/2}} + \\
 & \frac{1}{12(3+a)^2(4+a)^2(a-x(-8+8x-4x^2+x^3))^{5/2}} (a+8x-8x^2+4x^3-x^4)^{5/2} \\
 & \left(\left(40 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right. \\
 & \left. \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right)
 \end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right]},\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(46a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}}\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right]},\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(10 a^2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}\right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(112 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Big] + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \Big] \Big/ \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(32 a \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}}} \right) \\
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Big] + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& 28 \left(\left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) + \\
& \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) \right) - \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \right. \\
& \left. \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \Bigg/ \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2} \right] \right/ \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right) \right)$$

Problem 791: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 585 leaves, 15 steps):

$$\begin{aligned} & \frac{3}{8} (4+a) (1+(-1+x)^2) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} + \frac{1}{4} (1+(-1+x)^2) (3+a-2(-1+x)^2 - (-1+x)^4)^{3/2} + \\ & \frac{4(140+111a+21a^2)(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{315 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{2}{315} (2(80+27a) + 3(20+7a)(-1+x)^2) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) + \\ & \frac{1}{63} (15+7(-1+x)^2) (3+a-2(-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \frac{3}{8} (4+a)^2 \operatorname{ArcTan}\left[\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}\right] - \\ & \frac{4(140+111a+21a^2)(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{315 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\ & \frac{4(3+a)(100+33a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{315 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \end{aligned}$$

Result (type 4, 8500 leaves):

$$\frac{1}{a+8x-8x^2+4x^3-x^4} \left(\frac{1}{252} (404+107a) + \frac{(460+81a)x}{1260} - \frac{1}{360} (100+39a)x^2 + \frac{1}{315} (-80+77a)x^3 + \frac{71x^4}{42} - \frac{163x^5}{126} + \frac{19x^6}{36} - \frac{x^7}{9} \right) (a-x(-8+8x-4x^2+x^3))^{3/2} +$$

$$\frac{1}{1260 (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} (a - x (-8 + 8x - 4x^2 + x^3))^{3/2}$$

$$\left(- \left(\left(16160 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right.$$

$$\left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] \Big/$$

$$\left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) -$$

$$\left(5200 a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \\
& \left(162a^2\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(21280 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left. \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}\right], \right. \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(8016a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(-1-\sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right]\right], \\
& \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2} + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}\right], \\
& \operatorname{ArcSin}\left[\frac{\sqrt{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2} \Bigg] \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(546a^2 \left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right. \\
& \left. \left(-1-\sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right]\right], \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2} + 2\sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}, \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}\right]\right] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& 2240 \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right) + \right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}}\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\right) \right. \\
& \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}, \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}\right]\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) + \\
& \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) + \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 1776 a \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \right. \\
& \left. \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2} \right] \right) / \left(2\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right] \right), \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) + \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 336 a^2 \left((-1+\sqrt{-1-\sqrt{4+a}}+x) (-1-\sqrt{-1+\sqrt{4+a}}+x) (-1+\sqrt{-1+\sqrt{4+a}}+x) + \right. \\
& 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \right. \\
& \left. \left(- \left(-1-\sqrt{-1-\sqrt{4+a}} \right) \left(-2-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1+\sqrt{-1-\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right]\right], \\
& \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \Bigg/ \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) + \\
& \left(4\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right]\right], \right. \\
& \left.\left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \Bigg/ \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) \Bigg) \Bigg)
\end{aligned}$$

Problem 792: Result more than twice size of optimal antiderivative.

$$\int x^2 \sqrt{a+8x-8x^2+4x^3-x^4} dx$$

Optimal (type 4, 485 leaves, 13 steps):

$$\frac{1}{2} \left(1 + (-1+x)^2 \right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} + \frac{2(8+3a)(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{15 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} +$$

$$\frac{1}{15} \left(7+3(-1+x)^2 \right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{2} (4+a) \operatorname{ArcTan} \left[\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \right] -$$

$$\frac{2(8+3a)(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{15 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

$$\frac{8(3+a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{15 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

Result (type 4, 5647 leaves):

$$\left(\frac{1}{3} + \frac{x}{15} - \frac{x^2}{10} + \frac{x^3}{5} \right) \sqrt{a-x(-8+8x-4x^2+x^3)} + \frac{1}{15 \sqrt{a+8x-8x^2+4x^3-x^4}} \sqrt{a-x(-8+8x-4x^2+x^3)}$$

$$\left(- \left(\left(40 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right. \right.$$

$$\left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right.$$

$$\left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)} \right], \right.$$

$$\begin{aligned}
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left. \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) \right] - \\
& \left(2a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right] \right], \right. \\
& \left. \frac{\left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)} \right] / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} + 2 \sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \\
& \left(6a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right. \\
& \left. \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 16 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \right. \\
& \left. \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 6a \left((-1+\sqrt{-1-\sqrt{4+a}}+x) (-1-\sqrt{-1+\sqrt{4+a}}+x) (-1+\sqrt{-1+\sqrt{4+a}}+x) + \right. \\
& 2 \left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \right. \\
& \left(-\left(-1-\sqrt{-1-\sqrt{4+a}} \right) \left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) + \left(-1+\sqrt{-1-\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \right) \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)} \right], \right.
\end{aligned}$$

$$\left. \frac{\left(\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2}{\left(\frac{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}} \right)^2} \right] / \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) +$$

$$\left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}} \right)^2}{\left(\frac{\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}}{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right)$$

Problem 793: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}} dx$$

Optimal (type 4, 388 leaves, 11 steps):

$$\frac{(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \right] -$$

$$\frac{(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

$$\frac{\sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

Result (type 4, 1247 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a-x(-8+8x-4x^2+x^3)}} \\
& \left((-1+\sqrt{-1-\sqrt{4+a}}+x) (-1-\sqrt{-1+\sqrt{4+a}}+x) (-1+\sqrt{-1+\sqrt{4+a}}+x) + 2(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}) (-1-\sqrt{-1-\sqrt{4+a}}+x)^2 \right. \\
& \sqrt{\frac{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})(-1+\sqrt{-1-\sqrt{4+a}}+x)}{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})(1+\sqrt{-1-\sqrt{4+a}}-x)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}(-1-\sqrt{-1+\sqrt{4+a}}+x)}{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})(-1-\sqrt{-1-\sqrt{4+a}}+x)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}(-1+\sqrt{-1+\sqrt{4+a}}+x)}{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})(-1-\sqrt{-1-\sqrt{4+a}}+x)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} (\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})(-1+\sqrt{-1-\sqrt{4+a}}+x)}{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})(1+\sqrt{-1-\sqrt{4+a}}-x)}\right], \frac{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})^2}{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})^2}\right] + \right. \\
& \left. \left(-(-1-\sqrt{-1-\sqrt{4+a}})(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}) + (-1+\sqrt{-1-\sqrt{4+a}})(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})(-1+\sqrt{-1-\sqrt{4+a}}+x)}{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})(1+\sqrt{-1-\sqrt{4+a}}-x)}\right], \frac{(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}})^2}{(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}})^2}\right] \right) / \\
& \left(2\sqrt{-1-\sqrt{4+a}}(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}) \right) +
\end{aligned}$$

$$\left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \right. \right. \\ \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right)$$

Problem 794: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$$

Optimal (type 4, 311 leaves, 10 steps):

$$\frac{1 + (-1+x)^2}{(4+a) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(4+a) (2 + (-1+x)^2) (-1+x)}{2 (12+7a+a^2) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} - \\ \frac{(1-\sqrt{4+a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{2 (3+a) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{2 (3+a) \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}}$$

Result (type 4, 2941 leaves):

$$\frac{(-a-8x-ax+6x^2+ax^2-4x^3-ax^3)(a+8x-8x^2+4x^3-x^4)^2}{2(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)(a-x(-8+8x-4x^2+x^3))^{3/2}} - \frac{1}{2(3+a)(a-x(-8+8x-4x^2+x^3))^{3/2}} (a+8x-8x^2+4x^3-x^4)^{3/2} \\ \left(\left(2 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \right)$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)- \\
& \left(4\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(-1-\sqrt{-1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right],\right. \\
& \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]+2\sqrt{-1-\sqrt{4+a}}\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}\right],
\end{aligned}$$

$$\begin{aligned}
& \left. \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \Bigg/ \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) + \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& \left(\left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right), \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \left(\left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \left(-2 - \sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) + \left(-1 + \sqrt{-1-\sqrt{4+a}} \right) \right)
\end{aligned}$$

$$\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right],$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) +$$

$$\left(4 \text{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right)$$

Problem 795: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 582 leaves, 13 steps):

$$\begin{aligned}
& \frac{1 + (-1+x)^2}{3(4+a)(3+a-2(-1+x)^2 - (-1+x)^4)^{3/2}} + \frac{2(1+(-1+x)^2)}{3(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{(4+a)(2+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2 - (-1+x)^4)^{3/2}} + \\
& \frac{(29+7a+(13+3a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} - \frac{(13+3a)(1-\sqrt{4+a})(1+\frac{(-1+x)^2}{1-\sqrt{4+a}})(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{(13+3a)(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12(3+a)^2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12(12+7a+a^2)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}
\end{aligned}$$

Result (type 4, 5812 leaves):

$$\begin{aligned}
& \left((a+8x-8x^2+4x^3-x^4)^3 \right. \\
& \left. \left(\frac{a+8x+ax-6x^2-ax^2+4x^3+ax^3}{6(3+a)(4+a)(-a-8x+8x^2-4x^3+x^4)^2} + \frac{24-14a-6a^2-128x-36ax+84x^2+27a^2x^2+a^2x^2-52x^3-25ax^3-3a^2x^3}{12(3+a)^2(4+a)^2(-a-8x+8x^2-4x^3+x^4)} \right) \right) / \\
& (a-x(-8+8x-4x^2+x^3))^{5/2} - \frac{1}{12(3+a)^2(4+a)(a-x(-8+8x-4x^2+x^3))^{5/2}} (a+8x-8x^2+4x^3-x^4)^{5/2} \\
& \left(\left(20 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \right. \\
& \left. \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right]},\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \left(4a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}}\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right]},\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\right] \Big/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)-
\end{aligned}$$

$$\begin{aligned}
& \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \\
& \left(-1 - \sqrt{-1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} + 2 \sqrt{-1-\sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] \right) / \\
& \left(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \sqrt{a+8x-8x^2+4x^3-x^4} \right) - \\
& \left(12a \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right] \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4+a}} \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}} \right], \\
& \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Bigg] \Bigg/ \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 13 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \right. \\
& \left. \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \right) \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(2\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] \right) / \left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 3a \left((-1+\sqrt{-1-\sqrt{4+a}}+x) (-1-\sqrt{-1+\sqrt{4+a}}+x) (-1+\sqrt{-1+\sqrt{4+a}}+x) + \right. \\
& 2 \left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x \right)}} \left(\frac{1}{2\sqrt{-1-\sqrt{4+a}}} \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)} \right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right)^2} \right] + \right. \\
& \left(-\left(-1-\sqrt{-1-\sqrt{4+a}} \right) \left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) + \left(-1+\sqrt{-1-\sqrt{4+a}} \right) \left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \right) \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x \right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}} \right) \left(1+\sqrt{-1-\sqrt{4+a}}-x \right)} \right], \right.
\end{aligned}$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) +$$

$$\left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(1 + \sqrt{-1-\sqrt{4+a}} - x \right)} \right] \right),$$

$$\left. \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right)^2} \right] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \right) \right)$$

Problem 796: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$x^2 \sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29}(4+x)^2}{x^2} \right) \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{4+x}{\sqrt{3} 29^{1/4} x} \right], \frac{1}{58} (29 + \sqrt{29}) \right]$$

$$8 \sqrt{3} 29^{1/4} \sqrt{8+8x-x^3+8x^4}$$

Result (type 4, 927 leaves):

$$\begin{aligned}
& - \left(\left(2 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\left(\left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right)} \right] \right) \right. \right. \\
& \quad \left. \left(\left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right) \right) / \\
& \quad \left(\left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) / \\
& \quad \left(\left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right) \\
& \quad \left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right)^2 \\
& \quad \sqrt{\left(\left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \right) / \\
& \quad \left(\left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \right) \right) \\
& \quad \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \\
& \quad \sqrt{\left(\left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) / \\
& \quad \left(\left(x - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right)^2 \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right)^2 \right) \right) \right) / \\
& \quad \left(\sqrt{8 + 8x - x^3 + 8x^4} \left(-\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] + \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(\operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \operatorname{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right)
\end{aligned}$$

Problem 797: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 8x - x^3 + 8x^4)^{3/2}} dx$$

Optimal (type 4, 431 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(66 - \left(1 + \frac{4}{x}\right)^2\right) x^2}{1008 \sqrt{8 + 8x - x^3 + 8x^4}} + \frac{\left(216 - 7 \left(1 + \frac{4}{x}\right)^2\right) \left(1 + \frac{4}{x}\right) x^2}{12528 \sqrt{8 + 8x - x^3 + 8x^4}} + \frac{7 \left(261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4\right) \left(1 + \frac{4}{x}\right) x^2}{432 \sqrt{29} \sqrt{8 + 8x - x^3 + 8x^4} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)}
\end{aligned}$$

$$\frac{7x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right) \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{4+x}{\sqrt{3} 29^{3/4} x} \right], \frac{1}{58} \left(29 + \sqrt{29}\right) \right]}{144 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}} +$$

$$\frac{\left(14 - 5 \sqrt{29}\right) x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right) \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{4+x}{\sqrt{3} 29^{3/4} x} \right], \frac{1}{58} \left(29 + \sqrt{29}\right) \right]}{576 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}}$$

Result (type 4, 4865 leaves):

$$\begin{aligned}
& \frac{544 + 1539x - 1146x^2 + 784x^3}{21924\sqrt{8+8x-x^3+8x^4}} + \\
& \frac{1}{6264} \left(\left(28(x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) \right)^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \right.} \right. \\
& \quad \left. \left. \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \right. \\
& \quad \left. \left. \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right], - \left(\left((\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \right. \\
& \quad \left. \left((\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) / \left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \right. \right. \\
& \quad \left. \left. \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) \\
& \quad \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] + \text{EllipticPi} \left[\frac{-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]}{-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]}, \right. \\
& \quad \left. \text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) /} \right. \\
& \quad \left. \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right], \\
& \quad - \left(\left((\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, \right. \right. \\
& \quad \left. \left. 4]) \right) \right) / \left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \right. \\
& \quad \left. \left. \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) \left(-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] \right) \Big) \\
& \quad \sqrt{\left(\left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \right) /} \\
& \quad \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \\
& \quad \left(\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4] \right) \\
& \quad \sqrt{\left(\left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) /} \\
& \quad \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \\
& \quad \sqrt{\left(\left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) /} \\
& \quad \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \Big) \Big) / \\
& \quad \left(\sqrt{8+8x-x^3+8x^4} (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) \right) \\
& \quad \left(\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4] \right) \Big) + \\
& \quad \left(842 \text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) /} \right. \\
& \quad \left. \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) \Big), \\
& \quad \left((\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \Big) / \\
& \quad \left((\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \\
& \quad \left(\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4] \right) \Big) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2])^2 \\
& \quad \sqrt{\left(\left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \right) /} \\
& \quad \left((x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 3]) \right) \\
& \quad \left(\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4] \right) \\
& \quad \sqrt{\left(\left((-\text{Root}[8+8\#1-\#1^3+8\#1^4\&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4\&, 4]) \right) \right) /}
\end{aligned}$$

$$\left(\left(\left(\left(\left(x - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2\right]\right) \left(\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1\right] - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4\right] \right) \right) \right) \right) \right), \\ - \left(\left(\left(\left(\left(\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2\right] - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3\right] \right) \left(\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1\right] - \right. \right. \right. \right. \\ \left. \left. \left. \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4\right] \right) \right) \right) / \left(\left(-\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1\right] + \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3\right] \right) \right) \right. \\ \left. \left(\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2\right] - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4\right] \right) \right) \left(-\text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1\right] - \right. \\ \left. \left. \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2\right] - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3\right] - \text{Root}\left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4\right] \right) \right) \right) \right)$$

Problem 798: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + 4x + 4x^2 + 4x^4}} dx$$

Optimal (type 4, 108 leaves, 3 steps):

$$\frac{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10} \left(5 + \sqrt{5}\right)\right]}{2 \times 5^{1/4} \sqrt{1 + 4x + 4x^2 + 4x^4}}$$

Result (type 4, 249 leaves):

$$\left((2 - i) \sqrt{-\frac{1}{10} + \frac{i}{5}} \sqrt{\frac{(2i + \sqrt{-1 - 2i} - \sqrt{-1 + 2i})(-i + \sqrt{-1 - 2i} - 2x)}{(-2i + \sqrt{-1 - 2i} + \sqrt{-1 + 2i})(i + \sqrt{-1 - 2i} + 2x)}} \right. \\ \left. (1 + 2x + 2ix^2) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(2i + \sqrt{-1 - 2i} + \sqrt{-1 + 2i})(-i + \sqrt{-1 + 2i} + 2x)}{\sqrt{-1 + 2i}(i + \sqrt{-1 - 2i} + 2x)}}}{\sqrt{2}}}\right], \frac{1}{2} (5 - \sqrt{5})\right] \right) / \\ \left(\sqrt{\frac{(1 + 2i) \left((-1 + i) + \sqrt{-1 - 2i}\right) (1 + 2x + 2ix^2)}{(i + \sqrt{-1 - 2i} + 2x)^2}} \sqrt{1 + 4x + 4x^2 + 4x^4}} \right)$$

Problem 799: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1 + 4x + 4x^2 + 4x^4)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 9 steps):

$$\begin{aligned} & -\frac{\left(3 - \left(1 + \frac{1}{x}\right)^2\right) x^2}{\sqrt{1 + 4x + 4x^2 + 4x^4}} + \frac{\left(13 - 9\left(1 + \frac{1}{x}\right)^2\right) \left(1 + \frac{1}{x}\right) x^2}{10\sqrt{1 + 4x + 4x^2 + 4x^4}} + \frac{9\left(5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right) \left(1 + \frac{1}{x}\right) x^2}{10\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{1 + 4x + 4x^2 + 4x^4}} \\ & \frac{9\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10}\left(5 + \sqrt{5}\right)\right]}{2 \times 5^{3/4} \sqrt{1 + 4x + 4x^2 + 4x^4}} + \\ & \frac{3\left(3 - \sqrt{5}\right) \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10}\left(5 + \sqrt{5}\right)\right]}{4 \times 5^{3/4} \sqrt{1 + 4x + 4x^2 + 4x^4}} \end{aligned}$$

Result (type 4, 3334 leaves):

$$\begin{aligned} & \frac{19 + 42x - 16x^2 + 36x^3}{10\sqrt{1 + 4x + 4x^2 + 4x^4}} - \\ & \frac{3}{5} \left(- \left(\left(2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right)\right)}\right], \right. \right. \\ & \quad \left. \left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right) \right) \right), \\ & \quad \left(\left(\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right] - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 3\right]\right) \left(\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right) \right) / \left(\left(\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 3\right]\right) \right. \\ & \quad \left. \left(\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right] - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right) \right) \left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right)^2 \\ & \quad \sqrt{\left(\left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 3\right]\right)\right) / \\ & \quad \left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 3\right]\right)\right) \\ & \quad \left(\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right) \\ & \quad \sqrt{\left(\left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right)\right) / \\ & \quad \left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right)\right) \\ & \quad \sqrt{\left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right)\right) / \\ & \quad \left(\left(x - \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 4\right]\right)\right) \right) / \\ & \quad \left(\sqrt{1 + 4x + 4x^2 + 4x^4} \left(-\operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 1\right] + \operatorname{Root}\left[1 + 4\#1 + 4\#1^2 + 4\#1^4 \&, 2\right]\right) \right) \end{aligned}$$

Result (type 4, 6084 leaves):

$$\sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} \left(\frac{72888 + 89033x - 94314x^2 + 39280x^3}{241956(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} + \frac{65072399400 + 77274145879x - 83050578336x^2 + 34768831808x^3}{39028470624(8 + 24x + 8x^2 - 15x^3 + 8x^4)} \right) +$$

$$\frac{1}{78056941248} \left(\left(130383119280 (x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) \right)^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1])} \right. \right.$$

$$\left. \left. \frac{(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4])}{((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} \right), \right.$$

$$\left. - \left(\left(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3] \right) \right. \right.$$

$$\left. \frac{(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4])}{((-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} \right) \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] +$$

$$\text{EllipticPi} \left[\frac{-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]}{-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]}, \right.$$

$$\text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} / ((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))}],$$

$$\left. - \left(\left(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3] \right) \right. \right.$$

$$\left. \frac{(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4])}{((-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} \right) \right)$$

$$\left(-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] \right)$$

$$\sqrt{((-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3]))} / ((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 3])))$$

$$(\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4])$$

$$\sqrt{((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} / ((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4])))$$

$$\sqrt{((-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} / ((x - \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) (-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 4]))} /$$

$$\left(\sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} (-\text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 1] + \text{Root}[8 + 24\#1 + 8\#1^2 - 15\#1^3 + 8\#1^4 \&, 2]) \right)$$

$$\begin{aligned}
& - \frac{\left(176 - 23 \left(1 - \frac{6}{x}\right)^2\right) x^2}{51759 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \frac{\left(45401 - 3722 \left(1 - \frac{6}{x}\right)^2\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \frac{3722 \left(613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{3722 \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]}{51759 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} - \\
& \frac{\left(7444 - 145 \sqrt{613}\right) \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]}{207036 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}}
\end{aligned}$$

Result (type 4, 5428 leaves):

$$\begin{aligned}
& - \frac{2 \left(-106926 - 592639x + 232005x^2 + 44664x^3\right)}{10576089 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{1}{3525363} \left(\left(148880 \left(x - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right]\right)^2 \left(-\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right]\right)}\right)}\right] \right. \right. \right. \\
& \quad \left. \left. \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right] \right) \right) / \left(\left(x - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right]\right) \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right], \\
& \quad - \left(\left(\left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 3\right]\right) \right. \right. \\
& \quad \left. \left. \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right) / \\
& \quad \left(\left(-\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] + \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 3\right]\right) \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right) \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] + \\
& \quad \operatorname{EllipticPi}\left[\frac{-\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] + \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]}{-\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] + \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]}, \right. \\
& \quad \left. \operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right]\right) \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) / \left(\left(x - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right]\right) \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right] \right), \\
& \quad - \left(\left(\left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 3\right]\right) \right. \right. \\
& \quad \left. \left. \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right) / \\
& \quad \left(\left(-\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] + \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 3\right]\right) \left(\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right] - \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 4\right]\right) \right) \right) \right) \\
& \quad \left. \left(-\operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 1\right] + \operatorname{Root}\left[9 - 6\#1 - 44\#1^2 + 15\#1^3 + 3\#1^4 \&, 2\right]\right) \right)
\end{aligned}$$

Problem 819: Unable to integrate problem.

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Optimal (type 3, 45 leaves, 10 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Problem 820: Unable to integrate problem.

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Optimal (type 3, 45 leaves, 11 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 15 leaves):

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Problem 821: Unable to integrate problem.

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Optimal (type 3, 52 leaves, 12 steps):

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Problem 822: Unable to integrate problem.

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Optimal (type 3, 52 leaves, 13 steps):

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 17 leaves):

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Problem 843: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{1}{4}(-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{5+x} \text{ArcSinh}\left[\frac{\sqrt{-3+x}}{2\sqrt{2}}\right]}{\sqrt{-(-3+x)(5+x)}}$$

Problem 844: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(3-x)(5+x)}} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{1}{4}(-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{-3+x}}{2\sqrt{2}}\right]}{\sqrt{-(-3+x)(5+x)}}$$

Problem 846: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

`ArcSin[4 + x]`

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{-(3+x)(5+x)}}$$

Problem 847: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

`ArcSin[4 + x]`

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{-(3+x)(5+x)}}$$

Problem 852: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\frac{1}{-1+x^2}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \text{ArcSin}[x]$$

Result (type 3, 56 leaves):

$$\frac{1}{2} \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \left(-\text{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \text{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right] \right)$$

Problem 853: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\frac{1+x^2}{-1+x^4}} dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \text{ArcSin}[x]$$

Result (type 3, 56 leaves):

$$\frac{1}{2} \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \left(-\text{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \text{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right] \right)$$

Problem 855: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 11 leaves, 2 steps):

$$-2\sqrt{1-x}$$

Result (type 2, 23 leaves):

$$\frac{2(-1+x)\sqrt{1+x}}{\sqrt{1-x^2}}$$

Problem 857: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$2\sqrt{1+x}$$

Result (type 2, 25 leaves):

$$\frac{2\sqrt{1-x}(1+x)}{\sqrt{1-x^2}}$$

Problem 861: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx$$

Optimal (type 2, 11 leaves, 2 steps):

$$\frac{2}{3}(1+x)^{3/2}$$

Result (type 2, 27 leaves):

$$\frac{2(1+x)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Problem 863: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\sqrt{1+x}\sqrt{2+3x} - \frac{\text{ArcSinh}[\sqrt{2+3x}]}{\sqrt{3}}$$

Result (type 3, 79 leaves):

$$\frac{\sqrt{1-x} \left(3(1+x) \sqrt{2+3x} + \sqrt{3} \sqrt{-1-x} \operatorname{ArcTan} \left[\frac{\sqrt{3} \sqrt{-1-x}}{\sqrt{2+3x}} \right] \right)}{3 \sqrt{1-x^2}}$$

Problem 864: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2} x} dx$$

Optimal (type 3, 43 leaves, 7 steps):

$$\frac{4 \sqrt{1+x}}{\sqrt{1-x}} - \operatorname{ArcSin}[x] - \operatorname{ArcTanh}[\sqrt{1-x} \sqrt{1+x}]$$

Result (type 3, 101 leaves):

$$-\frac{4 \sqrt{1-x^2}}{-1+x} - 2 \operatorname{ArcSin} \left[\frac{\sqrt{1+x}}{\sqrt{2}} \right] + \operatorname{Log}[1 - \sqrt{1+x}] - \operatorname{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \operatorname{Log}[1 + \sqrt{1+x}] + \operatorname{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 866: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 7 steps):

$$\frac{4 \sqrt{1+ax}}{\sqrt{1-ax}} - \operatorname{ArcSin}[ax] - \operatorname{ArcTanh}[\sqrt{1-ax} \sqrt{1+ax}]$$

Result (type 3, 74 leaves):

$$\frac{4 \sqrt{1-a^2 x^2}}{1-ax} + \operatorname{Log}[x] - \operatorname{Log}[1 + \sqrt{1-a^2 x^2}] - i \operatorname{Log} \left[2 \left(-i ax + \sqrt{1-a^2 x^2} \right) \right]$$

Problem 869: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps):

$$\operatorname{ArcSin}[x]$$

Result (type 3, 32 leaves):

$$-\text{ArcTan}\left[\frac{x\sqrt{1+x^2}\sqrt{1-x^4}}{-1+x^4}\right]$$

Problem 871: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps):

$$\text{ArcSinh}[x]$$

Result (type 3, 42 leaves):

$$\text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}]$$

Problem 873: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$\frac{1}{2}x\sqrt{1-x^2} + \frac{\text{ArcSin}[x]}{2}$$

Result (type 3, 50 leaves):

$$\frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1+x^2}} + \text{ArcTan}\left[\frac{x\sqrt{1+x^2}}{\sqrt{1-x^4}}\right]\right)$$

Problem 875: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{1}{2}x\sqrt{1+x^2} + \frac{\text{ArcSinh}[x]}{2}$$

Result (type 3, 70 leaves):

$$\frac{1}{2} \left(\frac{x \sqrt{1-x^4}}{\sqrt{1-x^2}} + \text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}] \right)$$

Problem 911: Unable to integrate problem.

$$\int \frac{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}\right]}{\sqrt{2}\sqrt{b}}$$

Result (type 8, 39 leaves):

$$\int \frac{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Problem 912: Unable to integrate problem.

$$\int \frac{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}\right]}{\sqrt{2}\sqrt{b}}$$

Result (type 8, 40 leaves):

$$\int \frac{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Problem 913: Unable to integrate problem.

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)\sqrt{3 + 4x^4}} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \text{ArcTan}\left[\frac{\sqrt{3}d + 2icx}{\sqrt{2ic^2 - \sqrt{3}d^2}\sqrt{\sqrt{3} - 2ix^2}}\right]}{\sqrt{2ic^2 - \sqrt{3}d^2}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \text{ArcTanh}\left[\frac{\sqrt{3}d - 2icx}{\sqrt{2ic^2 + \sqrt{3}d^2}\sqrt{\sqrt{3} + 2ix^2}}\right]}{\sqrt{2ic^2 + \sqrt{3}d^2}}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)\sqrt{3 + 4x^4}} dx$$

Problem 914: Unable to integrate problem.

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2\sqrt{3 + 4x^4}} dx$$

Optimal (type 3, 268 leaves, 7 steps):

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(2ic^2 - \sqrt{3}d^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(2ic^2 + \sqrt{3}d^2)(c + dx)} + \frac{(1 + i)c \text{ArcTan}\left[\frac{\sqrt{3}d + 2icx}{\sqrt{2ic^2 - \sqrt{3}d^2}\sqrt{\sqrt{3} - 2ix^2}}\right]}{(2ic^2 - \sqrt{3}d^2)^{3/2}} + \frac{(1 - i)c \text{ArcTanh}\left[\frac{\sqrt{3}d - 2icx}{\sqrt{2ic^2 + \sqrt{3}d^2}\sqrt{\sqrt{3} + 2ix^2}}\right]}{(2ic^2 + \sqrt{3}d^2)^{3/2}}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2\sqrt{3 + 4x^4}} dx$$

Problem 918: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 + \frac{b}{x^2}}}{b + 2x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\frac{\text{ArcCsch}\left[\frac{\sqrt{2}x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 54 leaves):

$$\frac{\sqrt{2 + \frac{b}{x^2}} x \left(\text{Log}[x] - \text{Log}\left[b + \sqrt{b} \sqrt{b + 2x^2}\right] \right)}{\sqrt{b} \sqrt{b + 2x^2}}$$

Problem 919: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 - \frac{b}{x^2}}}{-b + 2x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\frac{\text{ArcCsc}\left[\frac{\sqrt{2}x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 64 leaves):

$$\frac{i \sqrt{2 - \frac{b}{x^2}} x \text{Log}\left[\frac{2(-i\sqrt{b} + \sqrt{-b + 2x^2})}{x}\right]}{\sqrt{b} \sqrt{-b + 2x^2}}$$

Problem 926: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}\left[\sqrt{2x+x^2}\right]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{2+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x(2+x)}}$$

Problem 927: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}\left[2\sqrt{x+x^2}\right]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{1+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{1+x}}\right]}{\sqrt{x(1+x)}}$$

Problem 929: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x-x^2}}{1+x} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$\sqrt{x-x^2} - \frac{3}{2}\text{ArcSin}[1-2x] + \sqrt{2}\text{ArcTan}\left[\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right]$$

Result (type 3, 120 leaves):

$$\frac{1}{2\sqrt{-1+x}\sqrt{x}}\sqrt{-(-1+x)x}\left(2\sqrt{-1+x}\sqrt{x} - 6\text{Log}[\sqrt{-1+x} + \sqrt{x}] + \sqrt{2}\text{Log}[1-2\sqrt{2}\sqrt{-1+x}\sqrt{x} - 3x] - \sqrt{2}\text{Log}[1+2\sqrt{2}\sqrt{-1+x}\sqrt{x} - 3x]\right)$$

Problem 951: Result unnecessarily involves higher level functions.

$$\int \frac{(1+\sqrt{x})^{1/3}}{x} dx$$

Optimal (type 3, 67 leaves, 6 steps):

$$6(1+\sqrt{x})^{1/3} - 2\sqrt{3}\text{ArcTan}\left[\frac{1+2(1+\sqrt{x})^{1/3}}{\sqrt{3}}\right] + 3\text{Log}\left[1 - (1+\sqrt{x})^{1/3}\right] - \frac{\text{Log}[x]}{2}$$

Result (type 5, 51 leaves):

$$\frac{6 + 6\sqrt{x} - 3\left(1 + \frac{1}{\sqrt{x}}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{1}{\sqrt{x}}\right]}{(1 + \sqrt{x})^{2/3}}$$

Problem 956: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right]}{\sqrt{b}\sqrt{d}}$$

Result (type 3, 99 leaves):

$$\frac{i\sqrt{a+bx}\sqrt{c-dx}\text{Log}\left[2\sqrt{a+bx}\sqrt{c-dx} - \frac{i(-bc+ad+2bdx)}{\sqrt{b}\sqrt{d}}\right]}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Problem 957: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x}(1-x^2)} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2}\text{Log}[1-\sqrt{x}] + \frac{1}{2}\text{Log}[1+\sqrt{x}]$$

Problem 958: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x}}{x-x^3} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 961: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}[\sqrt{2x+x^2}]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{2+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x(2+x)}}$$

Problem 969: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{x - \sqrt{1+2x^2}} dx$$

Optimal (type 3, 31 leaves, 7 steps):

$$-x - \sqrt{1+2x^2} + \text{ArcTan}[x] + \text{ArcTan}[\sqrt{1+2x^2}]$$

Result (type 3, 101 leaves):

$$\frac{1}{4} \left(-4x - 4\sqrt{1+2x^2} + 4\text{ArcTan}[x] - 4\text{ArcTan}\left[\frac{1}{\sqrt{1+2x^2}}\right] + 2i \text{Log}[1+x^2] - i \text{Log}[1+3x^2-2x\sqrt{1+2x^2}] - i \text{Log}[1+3x^2+2x\sqrt{1+2x^2}] \right)$$

Problem 981: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(1-x)x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcSin}[1-2x]$$

Result (type 3, 38 leaves):

$$\frac{2 \sqrt{-1+x} \sqrt{x} \operatorname{Log}[\sqrt{-1+x} + \sqrt{x}]}{\sqrt{-(-1+x)x}}$$

Problem 984: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1 + \sqrt{5-x^2} + \sqrt{5}x^2} dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTan}\left[\sqrt{\frac{1}{2}(3-\sqrt{5})x}\right]$$

Result (type 3, 39 leaves):

$$\frac{1}{4} i \operatorname{Log}[1 + \sqrt{5} - 2 i x] - \frac{1}{4} i \operatorname{Log}[1 + \sqrt{5} + 2 i x]$$

Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x} \sqrt{-1+x^2} dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} (3x + \sqrt{-1+x^2}) \sqrt{1-x^2+x} \sqrt{-1+x^2} + \frac{3 \operatorname{ArcSin}[x - \sqrt{-1+x^2}]}{4\sqrt{2}}$$

Result (type 8, 24 leaves):

$$\int \sqrt{1-x^2+x} \sqrt{-1+x^2} dx$$

Problem 996: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x+\sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}} dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} (\sqrt{x} + 3\sqrt{1+x}) \sqrt{-x + \sqrt{x} \sqrt{1+x}} - \frac{3 \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2\sqrt{2}}$$

Result (type 3, 180 leaves):

$$-\left(\left((1+x) (1+2x-2\sqrt{x}\sqrt{1+x})^2 \left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}} (-3-2x+2\sqrt{x}\sqrt{1+x}) + \right. \right. \right. \\ \left. \left. \left. 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \operatorname{Log}\left[2\sqrt{-x+\sqrt{x}\sqrt{1+x}} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \right] \right) \right) / \left(4(-\sqrt{x} + \sqrt{1+x})^3 (1+x - \sqrt{x}\sqrt{1+x})^2 \right) \right)$$

Problem 997: Unable to integrate problem.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{-2+\sqrt{5}}(x+\sqrt{1+x^2})\right] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{2+\sqrt{5}}(x+\sqrt{1+x^2})\right]$$

Result (type 8, 34 leaves):

$$-\int \frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Problem 998: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \operatorname{ArcTan}\left[\frac{2\sqrt{5} - (5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})}\sqrt{2+2x+x^2}}\right] - \sqrt{\frac{1}{2}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2\sqrt{5} + (5-\sqrt{5})x}{\sqrt{10(-1+\sqrt{5})}\sqrt{2+2x+x^2}}\right]$$

Result (type 3, 433 leaves):

$$\begin{aligned} & \frac{1}{4} \left(2 \sqrt{1+2i} \operatorname{ArcTan} \left[\left((8+8i) - (1-4i) x^3 + 5i \sqrt{1+2i} \sqrt{2+2x+x^2} + x^2 \left((-2+13i) + 5 \sqrt{1+2i} \sqrt{2+2x+x^2} \right) + \right. \right. \right. \\ & \quad \left. \left. \left. (1+i) x \left((9+5i) + 5 \sqrt{1+2i} \sqrt{2+2x+x^2} \right) \right) \right] / \left((4+14i) + (2+2i) x + (4-11i) x^2 - (3+8i) x^3 \right) \right] + \\ & 2i \sqrt{1-2i} \operatorname{ArcTanh} \left[\left((-8+8i) + (1+4i) x^3 + 5i \sqrt{1-2i} \sqrt{2+2x+x^2} + x^2 \left((2+13i) - 5 \sqrt{1-2i} \sqrt{2+2x+x^2} \right) + \right. \right. \\ & \quad \left. \left. (1+i) x \left((5+9i) + 5i \sqrt{1-2i} \sqrt{2+2x+x^2} \right) \right) \right] / \left((-14-4i) - (2+2i) x + (11-4i) x^2 + (8+3i) x^3 \right) \right] + \\ & i \left(\left(\sqrt{1-2i} - \sqrt{1+2i} \right) \operatorname{Log} [1+x^2] - \sqrt{1-2i} \operatorname{Log} \left[(7-4i) + (8-4i) x + (3-2i) x^2 + 4 \sqrt{1-2i} \sqrt{2+2x+x^2} + 2 \sqrt{1-2i} x \sqrt{2+2x+x^2} \right] + \right. \\ & \quad \left. \sqrt{1+2i} \operatorname{Log} \left[(7+4i) + (8+4i) x + (3+2i) x^2 + 4 \sqrt{1+2i} \sqrt{2+2x+x^2} + 2 \sqrt{1+2i} x \sqrt{2+2x+x^2} \right] \right) \end{aligned}$$

Problem 1002: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a + b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 184 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{a + b d^4 \left(\frac{c}{d} + x \right)^4}} \right]}{2 \sqrt{b} d^2} - \frac{c \left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a + b d^4 \left(\frac{c}{d} + x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} (c + d x)}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} b^{1/4} d^2 \sqrt{a + b d^4 \left(\frac{c}{d} + x \right)^4}}$$

Result (type 4, 330 leaves):

$$\left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \left(i \sqrt{a} + \sqrt{b} (c + d x)^2 \right) \right. \\ \left. \left(\left((-1)^{1/4} a^{1/4} - b^{1/4} c \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1 \right] - \right. \\ \left. \left. 2 (-1)^{1/4} a^{1/4} \text{EllipticPi} \left[-i, \text{ArcSin} \left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1 \right] \right) \right) / \\ \left(a^{1/4} \sqrt{b} d^2 \sqrt{\frac{i \sqrt{a} + \sqrt{b} (c + d x)^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} (c + d x) \right)^2}} \sqrt{a + b (c + d x)^4} \right)$$

Problem 1003: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 131 leaves, 2 steps):

$$\frac{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a + b d^4 \left(\frac{c}{d} + x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{b^{1/4} (c + d x)}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} b^{1/4} d \sqrt{a + b d^4 \left(\frac{c}{d} + x \right)^4}}$$

Result (type 4, 90 leaves):

$$\frac{i \sqrt{\frac{a + b (c + d x)^4}{a}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} (c + d x) \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{a + b (c + d x)^4}}$$

Problem 1004: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a + b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{bd-ae} x}{\sqrt{d} \sqrt{a+bx^2+cx^4}}\right]}{\sqrt{d} \sqrt{bd-ae}}$$

Result (type 4, 419 leaves):

$$\left(i \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] - \right. \right. \\ \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4ac}) d}{ae - \sqrt{a} \sqrt{-4cd^2 + ae^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] - \right. \\ \left. \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4ac}) d}{ae + \sqrt{a} \sqrt{-4cd^2 + ae^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \right) \right) / \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} d \sqrt{a + bx^2 + cx^4} \right)$$

Problem 1005: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a - b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{bd+ae} x}{\sqrt{d} \sqrt{a-bx^2+cx^4}}\right]}{\sqrt{d} \sqrt{bd+ae}}$$

Result (type 4, 416 leaves):

$$\left(i \sqrt{2 + \frac{4 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \left(\text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\ \left. \text{EllipticPi} \left[\frac{(b - \sqrt{b^2 - 4 a c}) d}{-a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] - \right. \\ \left. \left. \text{EllipticPi} \left[\frac{(-b + \sqrt{b^2 - 4 a c}) d}{a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] \right) \right) / \left(2 \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} d \sqrt{a - b x^2 + c x^4} \right)$$

Problem 1009: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e f - e f x^2}{(a d + b d x + a d x^2) \sqrt{a + b x + c x^2 + b x^3 + a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \text{ArcTan} \left[\frac{a b + (4 a^2 + b^2 - 2 a c) x + a b x^2}{2 a \sqrt{2 a - c} \sqrt{a + b x + c x^2 + b x^3 + a x^4}} \right]}{a \sqrt{2 a - c} d}$$

Result (type 4, 13884 leaves):

$$\frac{1}{d} e f \left(- \left(\left(8 (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2])^2 \left(\text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right]} \right) \right) / \left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right), \right. \\ \left. - \left(\left((\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) \right) \right. \right. \\ \left. \left. (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right) / \\ \left((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) - \\ 2 a \text{EllipticPi} \left[\left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) + \right. \right. \\ \left. \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right] / \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \right)$$

$$\int \frac{e f - e f x^2}{(-a d + b d x - a d x^2) \sqrt{-a + b x + c x^2 + b x^3 - a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \operatorname{ArcTanh}\left[\frac{a b - (4 a^2 + b^2 + 2 a c) x + a b x^2}{2 a \sqrt{2 a + c} \sqrt{-a + b x + c x^2 + b x^3 - a x^4}}\right]}{a \sqrt{2 a + c} d}$$

Result (type 4, 15147 leaves):

$$\begin{aligned} & \frac{1}{d} e f \left(- \left(\left(8 (x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \right)^2 \right. \right. \\ & \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]\right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right] / \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \right. \right. \\ & \left. \left. \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right)\right], \\ & - \left(\left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right) \right. \\ & \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right) / \\ & \left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, \right. \right. \\ & \left. \left. 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right) \left(b + \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]\right) - \\ & 2 a \operatorname{EllipticPi}\left[\left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \right. \right. \right. \\ & \left. \left. \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right) / \left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]\right) \right. \right. \\ & \left. \left. \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right], \\ & \operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]\right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \right. \right. \right. \\ & \left. \left. \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right] / \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \right. \\ & \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right], \\ & - \left(\left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right) \right. \\ & \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right) / \\ & \left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right) \right. \\ & \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right)\right) \\ & \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \\ & \sqrt{\left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \right. \\ & \left. \left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right)\right) / \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right) \right. \\ & \left. \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]\right)\right) \\ & \sqrt{\left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]\right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]\right)\right) / \\ & \left.\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]\right)\right) \end{aligned}$$

$$- \left(\left(x \sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a(-1+ax^2)}{b^2}} \right) \right. \right. \\ \left. \left. \left(\left[\text{Log} \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \text{Log} \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right] \right) / \left(\sqrt{2} \sqrt{\frac{a(-1+ax^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right) \right)^{3/2} \right) \right) \right)$$

Problem 1012: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-a x^2 + b x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{2} b \text{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\left(b^2 \sqrt{\frac{a(1+ax^2)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \sqrt{x \left(-ax + b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right)$$

$$\left(\left[\text{Log} \left[1 - \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] - \text{Log} \left[1 + \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] \right] \right) / \left(\sqrt{2} a^2 x \left(-1 - ax^2 + bx \sqrt{\frac{a(1+ax^2)}{b^2}} \right) \right)$$

Problem 1013: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x \left(ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \text{ArcSinh} \left[\frac{ax+b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 199 leaves):

$$- \left(\left(x \sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a(-1+ax^2)}{b^2}} \right) \right. \right. \\ \left. \left. \left(\left[\text{Log} \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \text{Log} \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) / \left(\sqrt{2} \sqrt{\frac{a(-1+ax^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a(-1+ax^2)}{b^2}} \right) \right)^{3/2} \right) \right)$$

Problem 1014: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x \left(-a x + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \text{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\left(b^2 \sqrt{\frac{a(1+ax^2)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \sqrt{x \left(-ax + b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right. \\ \left. \left(\left[\text{Log} \left[1 - \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] - \text{Log} \left[1 + \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] \right] \right) / \left(\sqrt{2} a^2 x \left(-1 - ax^2 + bx \sqrt{\frac{a(1+ax^2)}{b^2}} \right) \right) \right)$$

Problem 1015: Result more than twice size of optimal antiderivative.

$$\int \frac{-\sqrt{-4+x} - 4\sqrt{-1+x} + \sqrt{-4+x}x + \sqrt{-1+x}x}{(1 + \sqrt{-4+x} + \sqrt{-1+x})(4 - 5x + x^2)} dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$2 \text{Log} \left[1 + \sqrt{-4+x} + \sqrt{-1+x} \right]$$

Result (type 3, 75 leaves):

$$-\text{ArcTanh} \left[\sqrt{-4+x} \right] + \text{ArcTanh} \left[\frac{\sqrt{-1+x}}{2} \right] + \frac{1}{2} \text{Log} \left[17 - 4\sqrt{-4+x} - \sqrt{-1+x} - 5x \right] + \frac{1}{2} \text{Log} \left[5 - 2\sqrt{-4+x} - \sqrt{-1+x} - 2x \right]$$

Problem 1016: Unable to integrate problem.

$$\int \frac{1}{x(3+3x+x^2)(3+3x+3x^2+x^3)^{1/3}} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan} \left[\frac{1 + \frac{2 \cdot 3^{2/3}(1+x)}{(2+(1+x)^3)^{1/3}}}{\sqrt{3}} \right]}{3^{5/6}} - \frac{\text{Log} \left[1 - (1+x)^3 \right]}{6 \times 3^{1/3}} + \frac{\text{Log} \left[3^{1/3}(1+x) - (2 + (1+x)^3)^{1/3} \right]}{2 \times 3^{1/3}}$$

Result (type 8, 33 leaves):

$$\int \frac{1}{x (3 + 3x + x^2) (3 + 3x + 3x^2 + x^3)^{1/3}} dx$$

Problem 1017: Unable to integrate problem.

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - 2^{2/3}(1-x)}{\sqrt{3}(1-x^3)^{1/3}}\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[1 + 2(1-x)^3 - x^3\right]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log}\left[2^{1/3}(1-x) + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 8, 31 leaves):

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Problem 1018: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{-1 + x^4} (1 + x^4)} dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTan}\left[\frac{1 + x^2}{x \sqrt{-1 + x^4}}\right] - \frac{1}{4} \operatorname{ArcTanh}\left[\frac{1 - x^2}{x \sqrt{-1 + x^4}}\right]$$

Result (type 6, 114 leaves):

$$-\left(\left(7 x^3 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4\right]\right) / \left(3 \sqrt{-1 + x^4} (1 + x^4) \left(-7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4\right] + 2 x^4 \left(2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, x^4, -x^4\right] - \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, x^4, -x^4\right]\right)\right)\right)\right)$$

Problem 1019: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{(a e + c d x^2) (d + e x^2) \sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 3, 80 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c d^2 - b d e + a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a + b x^2 + c x^4}}\right]}{\sqrt{d} \sqrt{e} \sqrt{c d^2 - b d e + a e^2}}$$

Result (type 4, 383 leaves):

$$\left(i \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \right. \right. \\ \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) d}{2 a e}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \right. \\ \left. \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) e}{2 c d}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right] \right) / \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} d e \sqrt{a + b x^2 + c x^4} \right)$$

Problem 1021: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x + \sqrt{1 - x^2}} dx$$

Optimal (type 3, 122 leaves, 12 steps):

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}}{\sqrt{1-x^2}} x\right]}{\sqrt{3}}$$

Result (type 3, 2681 leaves):

$$\begin{aligned}
& \frac{(1+x\sqrt{1-x^2})\text{ArcSin}[x]}{x\left(\frac{1}{x}+\sqrt{1-x^2}\right)} + \left((-i+\sqrt{3}) \left(1+x\sqrt{1-x^2} \right) \right. \\
& \text{ArcTan}\left[\left(x \left(7i-\sqrt{3}+8i\sqrt{3}x+7ix^2+\sqrt{3}x^2 \right) \right) / \left(-6i+2\sqrt{3}+3x-11i\sqrt{3}x-18ix^2-2\sqrt{3}x^2-3x^3-3i\sqrt{3}x^3- \right. \right. \\
& \left. \left. 2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}-2i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2} \right) \right] / \left(2\sqrt{6(1-i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right) \right) - \\
& \left((-i+\sqrt{3}) \left(1+x\sqrt{1-x^2} \right) \text{ArcTan}\left[\left(x \left(7i-\sqrt{3}-8i\sqrt{3}x+7ix^2+\sqrt{3}x^2 \right) \right) / \left(6i-2\sqrt{3}+3x-11i\sqrt{3}x+18ix^2+2\sqrt{3}x^2- \right. \right. \right. \\
& \left. \left. \left. 3x^3-3i\sqrt{3}x^3+2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}+2i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2} \right) \right] \right) / \\
& \left(2\sqrt{6(1-i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right) \right) - \left((i+\sqrt{3}) \left(1+x\sqrt{1-x^2} \right) \text{ArcTan}\left[\left(x \left(-7i-\sqrt{3}-8i\sqrt{3}x-7ix^2+\sqrt{3}x^2 \right) \right) / \right. \right. \\
& \left. \left(-6i-2\sqrt{3}-3x-11i\sqrt{3}x-18ix^2+2\sqrt{3}x^2+3x^3-3i\sqrt{3}x^3-2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2}- \right. \right. \\
& \left. \left. \left. 2i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2}-2i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} \right) \right] \right) / \left(2\sqrt{6(1+i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right) \right) + \\
& \left((i+\sqrt{3}) \left(1+x\sqrt{1-x^2} \right) \text{ArcTan}\left[\left(x \left(-7i-\sqrt{3}+8i\sqrt{3}x-7ix^2+\sqrt{3}x^2 \right) \right) / \left(6i+2\sqrt{3}-3x-11i\sqrt{3}x+18ix^2-2\sqrt{3}x^2+ \right. \right. \right. \\
& \left. \left. \left. 3x^3-3i\sqrt{3}x^3+2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2}+2i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} \right) \right] \right) / \\
& \left(2\sqrt{6(1+i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right) \right) + \frac{i(-i+\sqrt{3})(1+x\sqrt{1-x^2})\text{Log}\left[(-i+\sqrt{3}-2x)^2(i+\sqrt{3}-2x)^2\right]}{4\sqrt{6(1-i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right)} - \\
& \frac{i(i+\sqrt{3})(1+x\sqrt{1-x^2})\text{Log}\left[(-i+\sqrt{3}-2x)^2(i+\sqrt{3}-2x)^2\right]}{4\sqrt{6(1+i\sqrt{3})}x\left(\frac{1}{x}+\sqrt{1-x^2}\right)} -
\end{aligned}$$

$$\frac{i(-i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [(-i + \sqrt{3} + 2x)^2 (i + \sqrt{3} + 2x)^2]}{4\sqrt{6(1-i\sqrt{3})} x \left(\frac{1}{x} + \sqrt{1-x^2}\right)} +$$

$$\frac{i(i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [(-i + \sqrt{3} + 2x)^2 (i + \sqrt{3} + 2x)^2]}{4\sqrt{6(1+i\sqrt{3})} x \left(\frac{1}{x} + \sqrt{1-x^2}\right)} -$$

$$\frac{i(1 + x\sqrt{1-x^2}) \operatorname{Log} \left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right]}{2\sqrt{3} x \left(\frac{1}{x} + \sqrt{1-x^2}\right)} +$$

$$\frac{i(1 + x\sqrt{1-x^2}) \operatorname{Log} \left[-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^2\right]}{2\sqrt{3} x \left(\frac{1}{x} + \sqrt{1-x^2}\right)} -$$

$$\left(i(-i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [3i + \sqrt{3} - 3x - 5i\sqrt{3}x + 10ix^2 + 3x^3 - 3i\sqrt{3}x^3 + ix^4 - \sqrt{3}x^4 + 2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2} - 3i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2} + 5i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2} - i\sqrt{6(1-i\sqrt{3})}x^3\sqrt{1-x^2}] \right) / \left(4\sqrt{6(1-i\sqrt{3})}x \left(\frac{1}{x} + \sqrt{1-x^2}\right) \right) +$$

$$\left(i(-i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [3i + \sqrt{3} + 3x + 5i\sqrt{3}x + 10ix^2 - 3x^3 + 3i\sqrt{3}x^3 + ix^4 - \sqrt{3}x^4 + 2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2} + 3i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2} + 5i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2} + i\sqrt{6(1-i\sqrt{3})}x^3\sqrt{1-x^2}] \right) / \left(4\sqrt{6(1-i\sqrt{3})}x \left(\frac{1}{x} + \sqrt{1-x^2}\right) \right) -$$

$$\left(i(i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [-3i + \sqrt{3} + 3x - 5i\sqrt{3}x - 10ix^2 - 3x^3 - 3i\sqrt{3}x^3 - ix^4 - \sqrt{3}x^4 - 2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2} - 3i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2} - 5i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} - i\sqrt{6(1+i\sqrt{3})}x^3\sqrt{1-x^2}] \right) / \left(4\sqrt{6(1+i\sqrt{3})}x \left(\frac{1}{x} + \sqrt{1-x^2}\right) \right) +$$

$$\left(i(i + \sqrt{3})(1 + x\sqrt{1-x^2}) \operatorname{Log} [-3i + \sqrt{3} - 3x + 5i\sqrt{3}x - 10ix^2 + 3x^3 + 3i\sqrt{3}x^3 - ix^4 - \sqrt{3}x^4 - 2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2} + 3i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2} - 5i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} + i\sqrt{6(1+i\sqrt{3})}x^3\sqrt{1-x^2}] \right) / \left(4\sqrt{6(1+i\sqrt{3})}x \left(\frac{1}{x} + \sqrt{1-x^2}\right) \right)$$

Problem 1022: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx$$

Optimal (type 3, 122 leaves, 13 steps):

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\frac{-i-\sqrt{3}}{i+\sqrt{3}}\sqrt{1-x^2}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}x}}{\sqrt{1-x^2}}\right]}{\sqrt{3}}$$

Result (type 3, 2155 leaves):

$$\text{ArcSin}[x] + \frac{1}{2\sqrt{6(1-i\sqrt{3})}}$$

$$\left(-i+\sqrt{3}\right) \text{ArcTan}\left[\left(x\left(7i-\sqrt{3}+8i\sqrt{3}x+7ix^2+\sqrt{3}x^2\right)\right)\right] / \left(-6i+2\sqrt{3}+3x-11i\sqrt{3}x-18ix^2-2\sqrt{3}x^2-3x^3-\right.$$

$$\left.3i\sqrt{3}x^3-2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}-2i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2}\right)] -$$

$$\frac{1}{2\sqrt{6(1-i\sqrt{3})}}\left(-i+\sqrt{3}\right) \text{ArcTan}\left[\left(x\left(7i-\sqrt{3}-8i\sqrt{3}x+7ix^2+\sqrt{3}x^2\right)\right)\right] / \left(6i-2\sqrt{3}+3x-11i\sqrt{3}x+18ix^2+\right.$$

$$\left.2\sqrt{3}x^2-3x^3-3i\sqrt{3}x^3+2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}+2i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2}\right)] -$$

$$\frac{1}{2\sqrt{6(1+i\sqrt{3})}}\left(i+\sqrt{3}\right) \text{ArcTan}\left[\left(x\left(-7i-\sqrt{3}-8i\sqrt{3}x-7ix^2+\sqrt{3}x^2\right)\right)\right] / \left(-6i-2\sqrt{3}-3x-11i\sqrt{3}x-18ix^2+\right.$$

$$\left.2\sqrt{3}x^2+3x^3-3i\sqrt{3}x^3-2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2}-2i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2}-2i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2}\right)] +$$

$$\frac{1}{2\sqrt{6(1+i\sqrt{3})}}\left(i+\sqrt{3}\right) \text{ArcTan}\left[\left(x\left(-7i-\sqrt{3}+8i\sqrt{3}x-7ix^2+\sqrt{3}x^2\right)\right)\right] / \left(6i+2\sqrt{3}-3x-11i\sqrt{3}x+18ix^2-\right.$$

$$\begin{aligned}
& \left. 2\sqrt{3}x^2 + 3x^3 - 3i\sqrt{3}x^3 + 2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2} - 2i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2} + 2i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2} \right] + \\
& \frac{i(-i+\sqrt{3})\operatorname{Log}\left[(-i+\sqrt{3}-2x)^2(i+\sqrt{3}-2x)^2\right]}{4\sqrt{6(1-i\sqrt{3})}} - \frac{i(i+\sqrt{3})\operatorname{Log}\left[(-i+\sqrt{3}-2x)^2(i+\sqrt{3}-2x)^2\right]}{4\sqrt{6(1+i\sqrt{3})}} - \\
& \frac{i(-i+\sqrt{3})\operatorname{Log}\left[(-i+\sqrt{3}+2x)^2(i+\sqrt{3}+2x)^2\right]}{4\sqrt{6(1-i\sqrt{3})}} + \\
& \frac{i(i+\sqrt{3})\operatorname{Log}\left[(-i+\sqrt{3}+2x)^2(i+\sqrt{3}+2x)^2\right]}{4\sqrt{6(1+i\sqrt{3})}} - \\
& \frac{i\operatorname{Log}\left[-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\right]}{2\sqrt{3}} + \\
& \frac{i\operatorname{Log}\left[-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^2\right]}{2\sqrt{3}} - \frac{1}{4\sqrt{6(1-i\sqrt{3})}} \\
& i(-i+\sqrt{3})\operatorname{Log}\left[3i+\sqrt{3}-3x-5i\sqrt{3}x+10ix^2+3x^3-3i\sqrt{3}x^3+ix^4-\sqrt{3}x^4+2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}-\right. \\
& \left. 3i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}+5i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2}-i\sqrt{6(1-i\sqrt{3})}x^3\sqrt{1-x^2}\right] + \frac{1}{4\sqrt{6(1-i\sqrt{3})}} \\
& i(-i+\sqrt{3})\operatorname{Log}\left[3i+\sqrt{3}+3x+5i\sqrt{3}x+10ix^2-3x^3+3i\sqrt{3}x^3+ix^4-\sqrt{3}x^4+2i\sqrt{2(1-i\sqrt{3})}\sqrt{1-x^2}+\right. \\
& \left. 3i\sqrt{6(1-i\sqrt{3})}x\sqrt{1-x^2}+5i\sqrt{2(1-i\sqrt{3})}x^2\sqrt{1-x^2}+i\sqrt{6(1-i\sqrt{3})}x^3\sqrt{1-x^2}\right] - \frac{1}{4\sqrt{6(1+i\sqrt{3})}} \\
& i(i+\sqrt{3})\operatorname{Log}\left[-3i+\sqrt{3}+3x-5i\sqrt{3}x-10ix^2-3x^3-3i\sqrt{3}x^3-ix^4-\sqrt{3}x^4-2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2}-\right.
\end{aligned}$$

$$\begin{aligned}
& 3 i \sqrt{6(1+i\sqrt{3})} x \sqrt{1-x^2} - 5 i \sqrt{2(1+i\sqrt{3})} x^2 \sqrt{1-x^2} - i \sqrt{6(1+i\sqrt{3})} x^3 \sqrt{1-x^2} \Big] + \frac{1}{4 \sqrt{6(1+i\sqrt{3})}} \\
& i(i+\sqrt{3}) \operatorname{Log}[-3i+\sqrt{3}-3x+5i\sqrt{3}x-10ix^2+3x^3+3i\sqrt{3}x^3-ix^4-\sqrt{3}x^4-2i\sqrt{2(1+i\sqrt{3})}\sqrt{1-x^2}+ \\
& 3i\sqrt{6(1+i\sqrt{3})}x\sqrt{1-x^2}-5i\sqrt{2(1+i\sqrt{3})}x^2\sqrt{1-x^2}+i\sqrt{6(1+i\sqrt{3})}x^3\sqrt{1-x^2}]
\end{aligned}$$

Problem 1024: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}} dx$$

Optimal (type 3, 177 leaves, 1 step):

$$\begin{aligned}
& \frac{1}{18432c^2} \operatorname{Log} [2073807360000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + \\
& 951050714480640000b^4c^8x^4 + 21641687369515008000b^3c^9x^5 + 32462531054272512000b^2c^{10}x^6 + \\
& 149587343098087735296c^{12}x^8 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4} \\
& (12203125b^6c^4 + 7920000b^5c^5x + 3888000b^4c^6x^2 + 110592000b^3c^7x^3 + 199065600b^2c^8x^4 + 12230590464c^{10}x^6)]
\end{aligned}$$

Result (type 4, 1671 leaves):

$$\begin{aligned}
& \left(2 \left(x - \frac{b \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2]}{c} \right)^2 \right. \\
& \left. \left(-\frac{1}{c} b \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\left((cx - b \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 1]) \right)} \right] \right) \right. \right. \\
& \left. \left. \left(\frac{\left(\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2] - \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 4] \right)}{\left((cx - b \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2]) \left(\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 1] - \right. \right. \right. \right.} \right. \\
& \left. \left. \left. \left. \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 4] \right) \right) \right) \right], \\
& - \left(\left(\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2] - \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 3] \right) \right. \\
& \left. \left(\frac{\left(\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 1] - \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 4] \right)}{\left(-\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 1] + \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 3] \right)} \right) \right. \\
& \left. \left(\operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2] - \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 4] \right) \right) \Big] \\
& \operatorname{Root}[-44375 + 576000\#1 + 576000\#1^2 + 5308416\#1^4, 2] + \frac{1}{c} \operatorname{EllipticPi} [
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1]}{c} + \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]}{c}}{\frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]}{c} + \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]}{c}}, \\
& \operatorname{ArcSin}\left[\sqrt{\left(\left(\left(c x - b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1]\right)\right.\right. \\
& \quad \left.\left.\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right) / \\
& \quad \left.\left(\left(c x - b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]\right)\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] - \right.\right.\right. \\
& \quad \left.\left.\left.\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right)], \\
& - \left(\left(\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 3]\right)\right.\right. \\
& \quad \left.\left.\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right) / \\
& \quad \left(\left(-\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] + \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 3]\right)\right. \\
& \quad \left.\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right) \\
& \left. \left(-b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] + b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]\right)\right) \\
& \sqrt{\left(\left(\left(-b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] + b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]\right)\right.\right. \\
& \quad \left.\left.\left(x - \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 3]}{c}\right)\right)\right) / \\
& \quad \left(c \left(x - \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]}{c}\right)\right) \\
& \quad \left(-\frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1]}{c} + \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 3]}{c}\right)\right) \\
& \sqrt{\left(\left(\left(c x - b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1]\right)\right.\right. \\
& \quad \left.\left.\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right) / \\
& \quad \left.\left(\left(c x - b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]\right)\right.\right. \\
& \quad \left.\left.\left(\operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] - \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]\right)\right)\right) \\
& \left(\frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1]}{c} - \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]}{c}\right) \\
& \sqrt{\left(\left(\left(-b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 1] + b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]\right)\right.\right. \\
& \quad \left.\left.\left(x - \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 4]}{c}\right)\right)\right) / \\
& \quad \left(c \left(x - \frac{b \operatorname{Root}[-44375+576000 \#1+576000 \#1^2+5308416 \#1^4, 2]}{c}\right)\right)
\end{aligned}$$

$$\left(\left(-\frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 1\right]}{c} + \frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 4\right]}{c} \right) \right) \Bigg/$$

$$\left(\sqrt{-44\,375 b^4 + 576\,000 b^3 c x + 576\,000 b^2 c^2 x^2 + 5\,308\,416 c^4 x^4} \right.$$

$$\left(-\frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 1\right]}{c} + \right.$$

$$\left. \frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 2\right]}{c} \right)$$

$$\left(\frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 2\right]}{c} - \right.$$

$$\left. \frac{b \operatorname{Root}\left[-44\,375 + 576\,000 \#1 + 576\,000 \#1^2 + 5\,308\,416 \#1^4 \&, 4\right]}{c} \right) \Bigg)$$

Problem 1025: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + 4x}{\sqrt{9 + 120x + 64x^2 + 64x^3 + 64x^4}} dx$$

Optimal (type 3, 100 leaves, 2 steps):

$$\frac{1}{16} \operatorname{Log}\left[921 + 2864x + 9280x^2 + 13440x^3 + 17024x^4 + 19456x^5 + 12288x^6 + 8192x^7 + 4096x^8 + \sqrt{9 + 120x + 64x^2 + 64x^3 + 64x^4} (179 + 444x + 744x^2 + 1280x^3 + 960x^4 + 768x^5 + 512x^6)\right]$$

Result (type 4, 2787 leaves):

$$\left(8(x - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2])\right)^2$$

$$\left(-\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 1]\right) \left(\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2] - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right)\right)}{\left(x - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2]\right)}\right] \right. \right.$$

$$\left. \left(\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 1] - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right)\right) \Bigg],$$

$$- \left(\left(\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2] - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 3]\right) \right.$$

$$\left. \left(\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 1] - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right)\right) \Bigg/$$

$$\left(\left(-\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 1] + \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 3]\right) \left(\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2] - \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right)\right) \Bigg] \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2] +$$

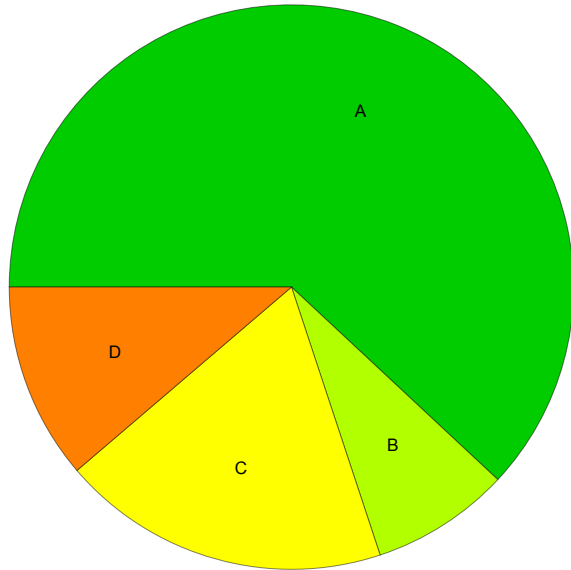
$$\operatorname{EllipticPi}\left[\left(-\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 1] + \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right) \Bigg/ \right.$$

$$\left. \left(-\operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 2] + \operatorname{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \&, 4]\right), \right]$$

$$\left(\sqrt{9 + 120x + 64x^2 + 64x^3 + 64x^4} \left(-\text{Root}\left[9 + 120t + 64t^2 + 64t^3 + 64t^4, 2\right] + \text{Root}\left[9 + 120t + 64t^2 + 64t^3 + 64t^4, 4\right] \right) \right)$$

Summary of Integration Test Results

1519 integration problems



A - 941 optimal antiderivatives

B - 121 more than twice size of optimal antiderivatives

C - 286 unnecessarily complex antiderivatives

D - 171 unable to integrate problems

E - 0 integration timeouts